Thrust augmentation optimization through supersonic after-burning in scramjet engine nozzles via surrogate-assisted evolutionary algorithms

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**Article Info**

**Abstract**

Scramjets are a class of hypersonic airbreathing engine that are associated with realizing the technology required for economical, reliable and high-speed access-to-space and atmospheric transport. The expanding flow in the scramjet nozzle comprises of unburned hydrogen which under ideal conditions, can be utilized to introduce an after-burning scheme. After-burning augments the thrust produced by the scramjet nozzle and creates a more robust nozzle design. This paper presents a single-objective design optimization considering three design variables with the objective of producing maximum thrust augmentation. It is found that significant levels of thrust augmentation are produced based upon contributions from increased pressure, mass flow and energy in the nozzle. Further understanding of the phenomenon by which thrust augmentation is being produced is provided in the form of variance-based global sensitivity analysis, force contribution breakdowns, analysis of the nozzle flowfields, analysis of the surface pressure and shear stress distributions acting on the nozzle wall and analysis of the combustion efficiency.

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1. Introduction

The development of hypersonic airbreathing engines is of great interest in space transportation as they provide an economically sound, reusable and high-speed platform for the transport of both civilians and cargo from the Earth to the low Earth orbit. Scramjet (supersonic combustion ramjet) propulsion is a promising hypersonic airbreathing technology which eliminates the need to carry an oxidizer and offers higher specific impulse than conventional rocket engines. Significant progress has been made in the development of scramjet technology over the last decade, with projects including NASA’s Hyper-X program [1] and the flight of the Boeing X-51A WaveRider in May 2010 [2]. A scramjet cycle demonstrated in Fig. 1, involves an intake of hypersonic air which is compressed to high pressure and temperature, fuel is then injected and combusted supersonically in the combustion chamber and the exhaust gas is expanded through the nozzle, resulting in net thrust.

The expanding exhaust gas comprises of a significant proportion of unburned hydrogen, which under ideal conditions can be combusted via the injection of oxygen directly into the unburned hydrogen stream, i.e., by introducing after-burning. This has the potential to significantly increase the thrust produced by the nozzle whilst also maintaining an ideal nozzle expansion ratio \( \frac{p_{\text{exit}}}{p_{\text{atm}}} \) by decreasing the injection pressure of oxygen as the flight altitude increases [3]. There are several critical factors that must be considered with such an after-burning scheme whereby the crossflow is in the supersonic and hypersonic...
range, including mixing of reactants, ignition and completion of combustion. Further, the phenomena that are associated with injection into the supersonic crossflow introduce several difficulties into the flowfield such as turbulent mixing, shock interaction and heat release [4–6].

There are various injector configurations that have been investigated for scramjet applications e.g., wall, ramp, and strut injectors which have varying impact on the efficiency of mixing and combustion that can be attained. A ramp-injector configuration allows efficient mixing with near streamwise injection which minimizes losses due to low-pressure gradients that occur downstream of shocks which are induced by an injection angle. The base of the ramp also provides a region for flame holding and flame stabilization through the buildup of a radical pool. However, the benefits of a ramp injector remain provided that the geometry does not result in too severe a local flow disturbance as this may result in pressure losses as well as more demanding wall cooling requirements [7–9]. A wall-injector with injection either upstream or downstream of a backward facing step introduces recirculation and provides a flame holding region with little or no pressure drop. It is also advantageous as the drag associated with flow separation over the cavity is less than that over a bluff body i.e., ramp-injector [10,11].

For the present study, the injection scheme combines a backward facing step with a ramp injector whereby a backward facing step is introduced to the nozzle wall with the fuel being injected from the face of the step (adapted from the ramp configuration). This hybrid injection scheme, as presented in Fig. 2 minimizes flow disturbances that may otherwise be present with ramp-injection whilst allowing versatility when introducing angular injection schemes.

Due to the extremely complex flowfields and highly coupled aerodynamic and aerothermal phenomena associated with the injection of fuel into the nozzle crossflow, it is imperative that optimum injection conditions are obtained, maximizing thrust augmentation and combustion efficiency. In order to achieve an optimization of this nature, an advanced design methodology, this couples a high fidelity computational fluid dynamics (CFD) code with evolutionary algorithms and the assistance of surrogate modeling. This state-of-the-art population-based technique can be applied to problems with either single or multiple design objectives. Surrogate modeling significantly reduces computational costs as the surrogate model replaces computationally expensive CFD simulations by approximating flowfields with mathematical formulations [12].

The scheme envisaged is effectively a combined-cycle scramjet rocket, or ejector scramjet. Whilst thrust is augmented substantially, specific impulse falls as the oxygen injection rate increases. Similar after-burning schemes have been proposed and/or tested for rocket engines. These include Supersonic After-Burning Rocket Engine (SABRE) [13] and Thrust Augmented Nozzle (TAN) [14–17]. TAN is a scheme introduced by GenCorp Aerojet in 2006, whereby numerical simulations and physical testing were conducted on a thrust augmented nozzle involving hydrogen–oxygen combustion upstream of the rocket nozzle throat. It was found that a significant level of thrust augmentation was achieved, attributed to increased mass flow, inertia and energy in the nozzle resulting from the TAN injection propellants. Further, they were able to eliminate thrust penalties due to over expansion of the nozzle. TAN considered the injection of both fuel and oxidizer in order to augment thrust; this led to efficient combustion adjacent to the nozzle wall upstream of the TAN injectors. If a similar injection scheme were to be introduced in scramjet after-burning it would eliminate the limitations associated with supersonic mixing and the inherent effect on combustion.

This paper builds upon research conducted by Ogawa and Boyce [18], who considered the design optimization of an axisymmetric scramjet nozzle for the SCRAMSPACE project conducted by The University of Queensland [19]. The optimized geometry acts as the baseline geometry for the present study. The optimization of the nozzle contour was based upon nozzle inflow conditions that were obtained from a separate CFD simulation in which the scramjet intake and combustor were included. The nozzle inflow therefore contained reacted gases and for the present study these nozzle inflow conditions remain unaltered.

The optimized geometry is adapted by including the injection of oxygen via the ramp-step configuration. The influence of several parameters on thrust augmentation is
investigated including the injection total pressure, streamwise injection position and injection angle. It is postulated that the streamwise injection position influences the mixing and combustion time significantly. Further, the position of the injector should be such that temperatures and pressures of the crossflow are sufficiently high, promoting sufficient fast combustion [6]. In addition, as the injection angle increases, enhanced penetration is expected to occur. However, higher injection angles lead to intensified levels of interactions between the injected oxygen and the crossflow, causing upstream and downstream flow separation and increased wall static temperatures. Finally it is postulated that the injection total pressure will influence the penetration levels of the fuel jet such that higher pressure will lead to enhanced penetration and allow for enhanced mixing and combustion. The momentum produced by the injected oxygen also directly influences the augmentation of thrust. The extremely complex nature of the scramjet nozzle flowfield means that it is imperative to optimize the aforementioned injection parameters in order to promote maximum thrust augmentation.

2. Methods

2.1. Conditions and configurations

2.1.1. Scramjet configuration
The axisymmetric scramjet engine being considered in this study is based on the SCRAMSPACE configuration [19]. The scramjet has 20 design parameters in total. Fig. 3 presents the configuration of the entire scramjet engine with all 20 parameters indicated to define the angle, length and curvature of the components. The optimization conducted by Ogawa and Boyce [18] considered the nozzle inlet radius fixed at 0.0351 m, which remains unaltered. Fig. 4 depicts the optimized nozzle geometry that is considered for this study.

2.1.2. Injection parameters
The injector is introduced to the geometry by translating the contour of the nozzle wall downstream of the injection point by the magnitude of the injector height \( h_j \) in the positive radial direction, creating a step configuration. Fig. 5 presents a parametric representation of the injection parameters considered for the optimization including the injection total pressure \( p_{j0} \), injection angle \( \theta_j \) and streamwise injection position \( x_j \). To implement a constant injection mass flow rate \( m_j \), the injector height \( h_j \) is considered as a function of \( p_j \) and \( x_j \) according to Eq. (1).

\[
h_j = \frac{m_j}{2\pi x_j \left( \frac{p_j}{p_{j0} \ T_j} \right)^{1/2}} \cdot \sqrt{\gamma R_{O_2} T_j}
\]

2.1.3. Freestream conditions
This study considers cruise conditions at an altitude of 27 km with the scramjet in operation at Mach 8. Table 1 shows the freestream conditions that are assumed, where the Reynolds number is based on an inlet capture radius of 0.075 m [18].

2.2. Oxygen enrichment
The fuel–air equivalence ratio for the expanding flow entering the nozzle \( \phi_{inlet} \) can be defined as the ratio of actual fuel mass to oxidizer mass entering the nozzle divided by the stoichiometric ratio of fuel mass to oxidizer mass. Assuming that the molar masses of H\(_2\) and O\(_2\) are 2 g and 32 g respectively and that the stoichiometric ratio required of H\(_2\):O\(_2\) for combustion is 2:1, the stoichiometric relationship of fuel mass to oxidizer mass can be defined as 1:8 meaning that the mass flow rate of oxygen must be eight times that of hydrogen as per Eq. (2).

\[
\phi_{inlet} = \frac{m_{fuel}/m_{O_2}}{(m_{fuel}/m_{O_2})_{stoichiometric}} = 8 \frac{m_{H_2}}{m_{O_2}} \frac{\gamma R_{O_2} T_j}{p_{j0} \ T_j}
\]

It is important to also quantify the level of oxygen enrichment in the nozzle due to oxygen injection in order to grasp a comprehensive understanding of the effect that different enrichment levels have on mixing and combustion in the nozzle. For this study the enrichment percentage \( EP \) is considered, defined as the mass flow rate of \( m_j \) as a percentage of the oxygen already present in the nozzle \( m_{O_2} \) that would be combusted if it were to react with the fuel.
REAMING H₂ IN THE NOZZLE [20,21].

\[ \text{EP} = \frac{100 \dot{m}_i}{\phi \dot{m}_{O_2}} = \frac{100 \dot{m}_i}{8 \dot{m}_{O_2}} \quad (3) \]

Finally, the oxygen enrichment in the scramjet nozzle can be considered according to the enriched equivalence ratio \( \phi_{en} \), which is the fuel–air equivalence ratio of flow entering the nozzle defined in Eq. (2), with the addition of the mass flow rate of the injected oxygen [21].

\[ \phi_{en} = 8 \cdot \frac{\dot{m}_{H_2}}{\dot{m}_{O_2} + \dot{m}_i} \quad (4) \]

A minimum enrichment percentage is selected for the present study such that the resultant mass flow rate of injected oxygen yields sufficient mixing and combustion [30]. Table 2 displays a summary of the enrichment parameters that are considered.

### Table 2
Oxygen enrichment parameters.

<table>
<thead>
<tr>
<th>EP [%]</th>
<th>( \phi_{en} )</th>
<th>( \dot{m}_i ) [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.159</td>
<td>3.6</td>
</tr>
</tbody>
</table>

been used in a wide range of problems concerning hypersonic aerodynamics [23,24]. Combustion within the nozzle is represented by the Evans and Schexnayder supersonic hydrogen flame model which considers 12 species and 25 chemical reactions as presented in Table 3 [25].

Both the nozzle and injector inflows are assumed to be fully turbulent and modeled with the two-equation SST \( k-\omega \) RANS model due to its robust modeling capabilities [26]. The solution is initialized from freestream conditions with the explicit under-relaxation factor reduced to 0.5. The CFL number is set to 0.5 for the first 100 iterations and then increased to 5, and run until a converged solution is obtained. The calculation is run for either 500 iterations, or until the energy residual dropped to \( 10^{-5} \) (whichever occurs first). Steady values for forces acting on the nozzle wall are observed after 500 iterations, even in cases where the energy residual is as high as \( 10^{-3} \).

### 2.3.2. Computational mesh

In building the two-dimensional structured mesh, it is ensured that the resolution is sufficiently fine to capture the boundary layers, shock waves and reactions accurately, whilst keeping computational time minimal. The geometry and meshes are both constructed using the ANSYS workbench tools Design Modeler and Fluent Meshing. A mesh convergence study which considers a balance between solution accuracy and computational cost is conducted for the baseline case and a final mesh resolution of 32,350 cells (32,806 nodes) is chosen as displayed in Fig. 6. This mesh resolution is used as a basis for the inclusion of the injector, with the actual resolution varying slightly depending on the injection configuration. The dimensionless wall distance \( y^+ \) ranges from 1.1 to 4.5 along the nozzle wall surface. The results of the study are presented in Section 2.3.4.

### 2.3.3. Flow and boundary conditions

The body of the scramjet is considered to comprise of isothermal cold walls at 300 K, which is valid for experiment in impulse facility or short duration flight testing.
Table 3
Species and reactions for the Evans and Schexnayder supersonic hydrogen combustion model.

<table>
<thead>
<tr>
<th>Species</th>
<th>Reactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H</td>
</tr>
<tr>
<td>2</td>
<td>O</td>
</tr>
<tr>
<td>3</td>
<td>H₂O</td>
</tr>
<tr>
<td>4</td>
<td>OH</td>
</tr>
<tr>
<td>5</td>
<td>O₂</td>
</tr>
<tr>
<td>6</td>
<td>H₂</td>
</tr>
<tr>
<td>7</td>
<td>N₂</td>
</tr>
<tr>
<td>8</td>
<td>N</td>
</tr>
<tr>
<td>9</td>
<td>NO</td>
</tr>
<tr>
<td>10</td>
<td>NO₂</td>
</tr>
<tr>
<td>11</td>
<td>HO₂</td>
</tr>
<tr>
<td>12</td>
<td>HNO₂</td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

[18]. The nozzle inflow profiles are obtained from CFD simulations for the nominal SCRAMSPACE specification [19] and are presented in Fig. 8. Pressure far-field conditions are imposed on the freestream, nozzle and injector inlets while the upper and downstream boundaries are set as pressure outlets as presented in Fig. 7. The downstream boundary is set more than 10 times the nozzle length downstream of the nozzle trailing edge in order to allow the flow to recover to freestream conditions. The pressure far-field condition is well suited to compressible flows and is used to model inlet conditions by Fluent with Mach number and static conditions being specified. The pressure outlet condition requires the input of static (gauge) pressure which is used only for subsonic flow. Once the flow become supersonic, the gauge pressure is no longer considered but rather the pressure is extrapolated from the interior flowfield [27].

2.3.4. Validation and mesh convergence

Initially a model is constructed incorporating the optimized geometry and nozzle inflow profiles obtained by Ogawa and Boyce [18] by using the commercial solver CFD++ to solve for the nozzle flowfields. This is done with the intention of replicating their findings and cross-validating to ensure that the solutions to the flowfields in ANSYS Fluent are consistent with the characteristics observed in the CFD++ results. Fig. 9 presents the total axial force acting on the nozzle and the surface pressure distributions as computed by Fluent and CFD++, it is demonstrated that there is a difference in total axial force of 3.4% validating that the scramjet nozzle is accurately modeled by Fluent. The small discrepancy between the two codes can be attributed to differences in meshes and computational methods. It can be seen that the static pressure along the nozzle wall is in reasonable agreement for both codes, validating that the scramjet nozzle is well modeled by Fluent. Fluent appears to have captured a more detailed representation of the flowfield characteristics than CFD++, which is demonstrated by the pressure rise due to the shock impingement just upstream of the nozzle throat and at x=0.9 m.

The total axial force acting on the scramjet nozzle is compared for three levels of mesh resolution. This comparison has been made prior to the introduction of the injector. The following resolutions are considered; coarse (32,350 cells), medium (133,911 cells) and fine (533,821 cells). For the medium resolution the mesh density in both axial and radial directions is increased by a factor of 2, for the fine resolution the mesh density in both radial and axial directions is increased by a factor of 4. Fig. 10 demonstrates that there is a minor difference between the three resolutions with a maximum deviation of 4%, thus for the present study the coarse resolution is considered.

2.4. Design optimization

2.4.1. Optimization algorithms

ANSYS Design Explorer [27] has the capabilities to perform Optimal Space-Filling Design (OSF) to populate...
the design space, generate a Kriging response surface and perform Genetic Algorithm (GA) based optimization; these capabilities are utilized to perform the optimization sequence outlined in the proceeding section. OSF builds an optimal set of design points based on pre-defined decision variables. OSF is essentially a Latin Hypercube Sampling Design (LHS) such that no two points will share a row of column of the design space. OSF differs from LHS as it incorporates post-processing to optimize the space between points through several iterations in order to achieve a comprehensive distribution across the design space [27]. A Kriging response surface is an interpolation method which models the design space based on a Gaussian process. It builds an accurate model of the design...
space which can fit higher order variations of the output parameter. The response surface provides a global model of the design space plus local deviations such that the Kriging model interpolates the OSF design points [27,28]. A Genetic Algorithm is a population based search heuristic which mimics the process of natural selection [29]. An initial population of 100 individuals is considered to initialize the optimization. A maximum of 20 generations with 100 individuals per generation are considered to define three candidates for a global optimum result. The convergence stability S is set to 2%.

2.4.2. Optimization sequence

The design optimization sequence incorporates Optimal Space-Filling Design [27] to generate design points based on true CFD evaluations, a Kriging algorithm to build a response surface and finally Genetic Algorithms (GA) to find a global optimum. 40 design points are generated initially, which involves the following steps; a computational mesh is generated in pre-processing for the design point (as selected by the OSF algorithm), the flowfield is then via a true CFD simulation and the solution forwarded to post-processing where the relevant data is extracted before the next OSF point is passed into meshing. Following this a Kriging algorithm is applied to the 40 design points in order to build a response surface. Finally Genetic Algorithms are employed in order to generate three candidate points for a global optimum. The optimization sequence is displayed in Fig. 11.

2.4.3. Optimization problem

The design optimization considers the three decision variables as specified in Section 2.1.2, the upper and lower bounds of which are selected based on a parametric study conducted on an identical baseline geometry with a similar after-burning scheme [30]. The sole objective of the optimization is to maximize the total axial force acting on the nozzle wall $F_x$ which is negative for drag and positive for thrust. The optimization problem statement for this study can be expressed as

Maximize: total axial force acting on the nozzle wall $F_x$ (N)
Subject to: injection angle $0^\circ \leq \theta_i \leq 45^\circ$
Injection total pressure $5 \text{ bar} \leq p_0 \leq 25 \text{ bar}$
Injector position $0.01 \text{ m} \leq x_i \leq 0.2 \text{ m}$

2.4.4. Sensitivity analysis

Variance-based global sensitivity analysis is performed in order to assess the impact of each decision variable (input) on the axial force acting on the nozzle wall $F_x$ (output). In particular, a numerical procedure based on Sobol’s variance decomposition [31] is employed to derive the sensitivity indices, based on surrogate prediction [32]. Input matrices $X$ of a base sample number of 10,000 and 3 columns for decision variables are built by using quasi-random numbers [33] within the range for each decision variable. Output vectors $Y$ are obtained by feeding the input matrices into the surrogate model with the best prediction accuracy. The first-order indices $S_i$ and total-effect indices $S_{ij}$ are calculated by the method outlined in Ref. [34], defined, respectively, as

$$S_i = \frac{\mathbb{V}(E(Y|X_i))}{\mathbb{V}(Y)}, \hspace{1cm} S_{ij} = 1 - \frac{\mathbb{V}(E(Y|X_{-i}))}{\mathbb{V}(Y)} \tag{5}$$

where $V$ and $E$ are the variance and conditional expected values, respectively.

3. Results

3.1. Design optimization

3.1.1. Objective functions

A GA-based optimization is undertaken in order to determine three candidates for a global optimum design configuration. Only the viscous and inviscid forces acting on the nozzle wall are considered for the optimization (defined in Eq. (8) in Section 3.2.4), while forces acting on the outer walls are included later as they do not change significantly based on the design parameters at hand [30]. The solution is found to converge to a stability percentage $S$ of 2% after 8 generations with a population of 800 individuals as presented in Fig. 12(a). Further, Fig. 12(b) presents the progression of $F_x$ as the population of individuals increases. Large gradients of $F_x$ can be observed in early generations which begin to become more stable after the first generation and finally converge after 8 generations. $S$ is calculated based on the mean of the i-th population $F_{xi}$, the standard deviation of the i-th population $\sigma(F_{xi})$ and the minimum and maximum values calculated in the first generated population $F_{x\text{ min}}$ and $F_{x\text{ max}}$ respectively [26]. The convergence criteria are considered fulfilled if the following two conditions are satisfied:

$$\left| \frac{F_{xi} - F_{x(i-1)}}{F_{x\text{ max}} - F_{x\text{ min}}} \right| < \frac{S}{100} \tag{6}$$

$$\left| \frac{\sigma(F_{xi}) - \sigma(F_{xi-1})}{F_{x\text{ max}} - F_{x\text{ min}}} \right| < \frac{S}{100} \tag{7}$$

Three candidate points are generated by the GA, which demonstrate total axial force acting on the nozzle wall deviating by less than 1%. The three generated candidate points display mixed levels of discrepancy between decision variables as presented in Table 4. It can be seen that the injection total pressure $p_0$ varies significantly, while injection angle $\theta_i$ shows some variance and little variance is seen in the streamwise injection position $x_i$. In order to validate the results of the optimization, CFD simulations are conducted for each candidate point as presented in Table 4. For the first two candidates the deviation is less than 1%, indicating that the 40 initial design points from true CFD simulations in combination.
with a Kriging response surface and a GA-based optimization are sufficient to interpolate the design space and gain an in-depth understanding of the scramjet flowfields. A greater deviation is observed for candidate 3, a possible reason for this may be that Candidate 3 seems to lie on the ridge of the peak $F_x$, where the direction of the optimization might not have been very decisive (as compared to the positions of the other candidates).

3.1.2. Design variables

The progression of the design parameters for all design points generated by the GA is presented in Fig. 13, in the form of a parallel coordinate plot. All values have been normalized according to their relevant global maximum. The black line demonstrates the global optimum and the dark lines which are in close vicinity to the optimum represent elites whose total axial force acting on the nozzle wall $F_x$ lie within 1% of the global optimum. All other lines represent the remaining design points generated by the Genetic Algorithm.

Parallel coordinate plots provide insight into the progressive selection of design variables and understanding of the evolutionary process, which was taken in order to determine the optimum configuration. Further they allow for the determination of primary and secondary parameters along with the sensitivity analysis, which is to be presented in the following section. The overlapping of lines is indicative of primary parameters that play a crucial role in achieving optimum performance. The primary design parameter appears to be the streamwise injection position $x_j$ whereby the elite design points show minimal deviation from the optimum. The injection angle $\theta_j$ is a secondary parameter followed by the injection total pressure $p_{0j}$, which demonstrates little overlapping of elite design points, indicating that thrust augmentation is least sensitive to this parameter.

3.1.3. Sensitivity analysis

The impact of the design variables $p_{0j}$, $x_j$ and $\theta_j$ on the objective function $F_x$ has been investigated through conducting a variance-based global sensitivity analysis which uses surrogate models trained with CFD-evaluated solutions as
described in Section 2.4.4. The first order $S_i$ and total-effect $S_{Ti}$ indices for the total axial force acting on the nozzle wall are plotted in Fig. 14. It can be seen that both the first order and total effect indices are characterized by the sum of the indices being unity i.e., $\Sigma S_i = 1$ and $\Sigma S_{Ti} = 1$, indicating that the effects of the individual design variables $p_{0j}$, $x_j$, and $\theta_j$ on the objective function $F_x$ are linearly additive and the total effects of the design variables on $F_x$ is the summation of the effects of the individual variables. Any difference between the first-order index and total effect index is indicative of the effect that the design variable has on the objective function when interacting with other design variables [33]. It can be seen that the injection angle has greater values of $S_{Ti}$ than $S_i$ indicating that it is more actively involved in interactions than the other parameters.

![Fig. 14. Sensitivity indices on the total axial force acting on the nozzle wall.](image)

![Fig. 15. Contour plots of total axial force acting on the nozzle wall.](image)
3.1.4. Force contour plots

The total axial force acting on the nozzle wall \( F_x \) is found to be most sensitive to the streamwise injection position \( x_j \). The injection angle \( \theta_j \) is found to have some impact whereas the injection total pressure \( p_{0j} \) exerts relatively insignificant impact. To further investigate the roles of the decision variables, variations of \( F_x \) are demonstrated in the form of force contour plots as presented in Fig. 15. The contour plots are derived by applying surface fitting methods to the surrogate predictions generated by the Kriging response surface. The contours in Fig. 15(a) are of a vertical nature indicating that the total axial force acting on the nozzle wall \( F_x \) has a greater dependence on the injection angle \( \theta_j \) than the injection total pressure \( p_{0j} \). The contours in Fig. 15(b) are vertical with some variance, indicating that the total axial force acting on the nozzle wall \( F_x \) is primarily dependent on \( x_j \) with some secondary dependence on the injection angle \( \theta_j \). Finally, the contours in Fig. 15(c) are vertical with little variance, indicating that the total axial force acting on the nozzle wall \( F_x \) is relatively insensitive to the injection total pressure \( p_{0j} \) and therefore variance \( p_{0j} \) does not contribute significantly to \( F_x \). These findings further support those of the two preceding sections.

3.2. Analysis

3.2.1. Total axial force

The total axial force \( F_{xt} \) acting on the nozzle is calculated including the contribution of viscous and inviscid forces acting on the nozzle and exterior walls. A breakdown of the optimum injection configuration and a force comparison with the baseline model are presented in Table 5.

It is found that the streamwise injection position \( x_j \) is the dominating factor when considering the level of attainable thrust augmentation in such a manner that the level of thrust augmentation decreases as the injector moves downstream of the nozzle throat. This can be attributed to the fact that less of the nozzle wall is exposed to the pressure increase caused by the after-burning scheme, as is to be further examined in Section 3.2.4. Table 5 presents the combination of parameters which are found to be optimum, inducing the highest total axial force in comparison with the baseline geometry.

It is found that the greatest level of thrust augmentation is obtained at an injection angle of \( \theta_j = 23.5^\circ \). This is at a streamwise position where the contour of the nozzle wall has an angle of \( \theta_{\text{nozzle}} = 7.7^\circ \) leading to an injection angle from the horizontal of \( \alpha_j = 15.8^\circ \). For injection angles less than this sufficient penetration into the crossflow does not occur and for steeper injection angles the losses incurred due to intensified levels of interaction between the fuel jet and the crossflow causing upstream and downstream flow separation become too significant and surpass the enhanced levels of penetration.

It is found through the variance based global sensitivity analysis and inspection of the design points that the injection total pressure does not have significant impact on thrust augmentation. This contradicts the findings of Candon et al. [30] where the injection total pressure was found to be the main contributing factor to thrust augmentation for a similar after-burning scheme adapted to the same baseline geometry. The defining discrepancy between the two studies is that the injection mass flow rate \( m_j \) was not fixed in the preceding study therefore the injection mass flow rate increased significantly with increasing injection total pressure. This indicates that provided the injection mass flow rate is constant, the pressure increase on the nozzle wall downstream of the injection point is not significantly affected by the injection total pressure and that thrust augmentation is highly sensitive to the injection mass flow rate.

3.2.2. Force breakdown

In order to determine the source of axial forces, Figs. 16–18 present a breakdown of the inviscid, viscous and jet force contributions, comparing the baseline geometry with the optimum injection configuration and six other nominal design points. The nominal cases are identical to the optimum injection configuration with the variance of one design parameter. The total axial force \( F_{xt} \) is significantly dominated by the inviscid force acting on the nozzle wall (thrust). There is also a substantial contribution to thrust augmentation that can be attributed to the momentum increase caused by the fuel jet and a small contribution attributed to the inviscid force acting on the nozzle base. Whilst viscous (drag) forces on the nozzle and freestream walls are present, they are significantly outweighed by the inviscid forces.

Fig. 16 demonstrates that as the injection total pressure \( p_{0j} \) varies significantly, the axial forces acting on the nozzle wall remain relatively unchanged. Further, Figs. 17 and 18 demonstrate that the streamwise injection position \( x_j \) has a significant impact on axial forces acting on the nozzle wall whilst the injection angle \( \theta_j \) has moderate impact. These findings provide further evidence that the thrust increase due to aerodynamic phenomena, namely the pressure increase within the nozzle, is not affected by the injection total pressure \( p_{0j} \) provided that the injection mass flow rate \( m_j \) remains constant, it also demonstrates that thrust augmentation is most sensitive to the streamwise injection position. It is apparent that a significant portion of the thrust augmentation is attributed to the momentum increase caused by the fuel jet. In practice, however, the losses incurred due to the storage of oxygen and the inclusion of high pressure pumps to achieve this momentum are likely to counteract the net gain in thrust and thus it is likely that the net thrust increase is not as prominent as indicated. Addressing and quantifying this matter is a prospect for future work.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Optimum</th>
<th>Baseline (injection off)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_j ) [m]</td>
<td>0.01067</td>
<td>–</td>
</tr>
<tr>
<td>( \theta_j ) [°]</td>
<td>23.55</td>
<td>–</td>
</tr>
<tr>
<td>( p_{0j} ) [bar]</td>
<td>24.37</td>
<td>–</td>
</tr>
<tr>
<td>( F_{xt} ) [N]</td>
<td>2075</td>
<td>554</td>
</tr>
</tbody>
</table>
3.2.3. Nozzle flowfields

Flowfields for the Mach number $M$ and the mass fraction of water $CH_2O$ as a product of combustion are visualized in Figs. 19–21, comparing the cases of baseline geometry with the optimum injection configuration and six other nominal cases. The nominal cases are identical to the optimum injection configuration with the variance of one design parameter. Observation of the water mass fraction distributions indicates that the oxygen jet is achieving substantial penetration into the crossflow leading to combustion as indicated by the production of $H_2O$. It is noteworthy that whilst injection total pressure $p_{0j}$ does not have a significant effect on thrust augmentation, it does on mixing and combustion efficiencies. This is presented in Fig. 19(a), where it can be seen that at higher injection total pressures more $H_2O$ is produced, indicative of enhanced combustion which can be attributed to intensified levels of penetration caused by the high-pressure fuel jet. The physical reason behind this not contributing to thrust augmentation is presented in Section 3.2.4. Fig. 20(a) indicates that the streamwise injection position $x_j$ significantly affects combustion such that as $x_j$ moves downstream of the nozzle throat the production of water diminishes, this is attributed to reduced static temperatures and pressures that occur within the nozzle as the flow expands. Fig. 21(a) demonstrates that as the injection angle $\theta_j$ increases the combustion efficiency decreases, indicating that increased injection angles are not beneficial in consideration of mixing and combustion efficiencies. This unexpected finding is attributed to the axisymmetric flowfield ignoring streamwise vorticity which can be produced by injecting a jet into a crossflow. Hence, greater injection angles introduce more prominent streamwise vortices which inherently enhances mixing. However, as streamwise vortices are neglected in the present simulations, the advantages of increasing the injection angle are not fully realized.

The Mach number flowfield for the optimum injection configuration demonstrate the considerable effects of oxygen injection on the crossflow where a prominent bow shock is formed due to interactions between the fuel jet and crossflow followed by expansion in the separated region downstream. Reflection of the shock can be observed at approximately $x=0.05$ m followed by a reimpingement on the nozzle wall at $x=0.11$ m. A second reflection of the shock can be observed at $x=0.33$ m before exiting the nozzle in the freestream. Fig. 19(b) indicates that as the injection total pressure decreases the effect of the fuel jet on the crossflow is less significant whereby the bow shock that is formed in the injection region becomes far less prominent. Consequently the reflection of the shock and reimpingement on the nozzle wall are also less prominent. Fig. 20(b) demonstrates that as the streamwise injection position moves downstream of the nozzle throat the shock structure caused by the interaction between the fuel jet and crossflow becomes more intense. This can be attributed to the increased velocity of the crossflow that occurs as the flow expands.

Whilst combustion is observed, the phenomenon is not as significant as expected. This is assumed to be attributed to inadequate diffusion between the fuel jet and crossflow which is a result of the highly compressible nature of the supersonic/hypersonic crossflow. Further, the axisymmetric nature of the model ignores streamwise vortices which would likely contribute to the enhancement of mixing [35].

3.2.4. Surface forces

The axial forces acting on the nozzle wall are attained by integrating the pressure, $p$, and shear stress, $\tau_x$, as presented in following equation:

$$F_x = F_{x\text{ inviscid}} + F_{x\text{ viscous}}$$

$$= \int_{r_{wall}}^{r_{exit}} 2\pi r p dr + \int_0^{4\pi} 2\pi r \tau_x dx$$

Fig. 16. Force breakdown of axial force components comparing the baseline and optimum cases with optimum cases that vary in injection total pressure $p_{0j}$.

Fig. 17. Force breakdown of axial force components comparing the baseline and optimum cases with optimum cases that vary in streamwise injection position $x_j$.

Fig. 18. Force breakdown of axial force components comparing the baseline and optimum cases with optimum cases that vary in injection angle $\theta_j$. 


\[ \frac{1}{2} \pi r \int_{r_{\text{exit}}}^{r_{\text{inlet}}} p \, dr + \int_0^{l_n} \tau_s \, dx \]  

where, \( r_{\text{inlet}}, r_{\text{exit}} \) and \( l_n \) represent the nozzle inlet radius, nozzle outlet radius and nozzle length, respectively.

The surface pressure and shear stress distributions for the baseline and optimum configurations are compared with six nominal design points as presented in Figs. 22–24. The nominal cases are identical to the optimum injection configuration with the variance of one design parameter.
It can be seen that the optimum injection configuration exhibits a significant increase in surface pressure in the region of the injection point. This is followed by a drop in surface pressure downstream due to separation and expansion then an increase, which can be attributed to the reimpingement of the reflected shock on the nozzle wall. Finally the shock exits the nozzle in the freestream and the pressure stabilizes.

Fig. 22 demonstrates that as the injection total pressure \( p_0 \) drops, the surface pressure increase in the injection and reimpingement regions both reduce, however, the pressure drop in the separated region can also be seen to reduce. These occurrences counteract each other, which elucidates the physical reason that the injection total pressure does not significantly impact thrust augmentation.

Observation of Fig. 23 indicates that as the streamwise injection position moves downstream, the pressure increase in the injection region and due to the reimpingement of the shock on the nozzle wall diminishes. This can be attributed to the increased crossflow Mach numbers which occur as injection moves downstream.

It can be seen in Fig. 24 that at a greater injection angle \( \theta_j \) the thrust increase in the injection region is reduced, however, at a lower injection angle the thrust increase in the injection region is constant with the optimum configuration. The pressure rise due to shock reimpingement can be seen to reduce for injection angles greater or less than the optimum and for the lower injection angle, reimpingement can be seen to move slightly downstream.

The shear stress distribution for the optimum configuration demonstrates similar behavior to the surface pressure distributions with an increase in the region of the injection point followed by a decrease in the separated region downstream before stabilizing. Further, as the design parameters are varied the shear stress distributions continue to demonstrate similar behavior to the surface pressure distributions. The shear stress and surface pressure distributions are consistent with the shock structures that can be observed in the Mach number distributions in Section 3.2.4. The main source of thrust augmentation appears to be the extreme surface pressure increases that are observed in the injection region and where the reflected shock reimpinges on the nozzle wall. Whilst the increase in shear stress on the nozzle is found to be significant, it is also negligible as the increase in surface pressure is by far dominant.

### 3.2.5 Combustion

In order to quantify the combustion efficiency, the mass fractions of hydrogen \( C_H \) and water \( C_{H_2O} \) at the nozzle outlet are compared for the optimum configuration for identical simulations that differ only by the presence of chemical reactions as presented in Fig. 25. Without chemical reactions the species are being transported through the nozzle without reacting hence the difference between the two simulations can be considered a result of combustion. A decrease in hydrogen and an increase in water can be observed indicating that the after-burning scheme is generating combustion. As presented in Table 6, less than 10% of the remaining hydrogen is being combusted for the optimum injection configuration. This observation gives reason for further optimization studies in which a tradeoff between thrust augmentation and combustion efficiency should be considered, with primary focus on the thrust increase as a result of only combustion within the nozzle.
4. Conclusion

A numerical study has been conducted to investigate thrust augmentation through supersonic afterburning in axisymmetric scramjet nozzles. A design optimization has been performed to examine the effects of different configurations represented by three design parameters, namely, the streamwise injection position, injection total pressure and injection angle, on thrust augmentation. Evolutionary algorithms with the assistance of surrogate modeling have been employed, coupled with computational fluid dynamics. The results are presented in the form of sensitivity analysis, force contour plots, axial force breakdown, and pressure distributions in order to gain insight into the effects of the design parameters on thrust augmentation and in-depth understanding of the nozzle flowfield structure that is associated with the inclusion of the after-burning scheme.

For the optimum injection configuration the total thrust acting on the nozzle was found to increase considerably in comparison to the baseline geometry and the thrust augmentation is primarily affected by the streamwise injection position with secondary contributions from the injection angle and insensitivity to the injection total pressure provided that the injection mass flow rate is fixed.

The highly compressible nature of the supersonic/hypersonic crossflow presents a situation in which mixing of the fuel jet with the crossflow becomes increasingly challenging. Consequently, combustion of the unburned hydrogen present in the scramjet nozzle is not as significant as expected.
The complex phenomena associated with the aerodynamics and chemical reactions in the scramjet nozzle introduced a scenario where an optimum configuration requires a fine balance between the parameters that are investigated in this paper. Future work will include an investigation into the enhancement of mixing and combustion through design optimizations which consider maximizing the thrust increase attributed to combustion only and maximizing the total amount of hydrogen burned. This will be followed by a design optimization considering a strut injection scheme focusing on minimizing the energy associated with fuel injection whilst quantifying and maximizing the thrust increase due to combustion.

Acknowledgments

The authors thank Dr. Robert Carrese from LEAP Australia for his support with regard to the ANSYS software package.

Appendix A. Nomenclature

\[ \begin{align*} 
\alpha &= \text{angle from horizontal [°]} \\
C &= \text{mass fraction} \\
E &= \text{conditional expected value} \\
EP &= \text{enrichment percentage [%]} \\
Fx &= \text{total axial force acting on the nozzle wall [N]} \\
fX &= \text{axial force component [N]} \\
F_{ax} &= \text{total axial force acting on the nozzle [N]} \\
\gamma &= \text{specific heat ratio} \\
h &= \text{radial height [m]} \\
H &= \text{altitude [km]} \\
l_n &= \text{nozzle length [m]} \\
m &= \text{mass flow rate [kg/s]} \\
M &= \text{Mach number} \\
p &= \text{static pressure [Pa]} \\
p_0 &= \text{total pressure [bar]} \\
\phi_{inlet} &= \text{fuel–air equivalence ratio} \\
\phi_{en} &= \text{enriched fuel–air equivalence ratio} \\
q &= \text{dynamic pressure [kPa]} \\
r &= \text{radius or radial coordinate [m]} \\
R &= \text{ideal gas constant [J/kg K]} \\
Re &= \text{Reynolds number} \\
S &= \text{convergence stability [%]} \\
\sigma &= \text{standard deviation} \\
S_i &= \text{first-order sensitivity index of } x_i \\
S_{ti} &= \text{total-effect sensitivity index of } x_i \\
T &= \text{static temperature [K]} \\
\tau_x &= \text{axial shear stress [N/m]} \\
\theta &= \text{total angle [°]} \\
u &= \text{velocity in x-direction [m/s]} \\
V &= \text{variance} \\
x &= \text{axial coordinate [m]} \\
X &= \text{input matrix for sensitivity analysis} \\
Y &= \text{output vector for sensitivity analysis} \\
\infty &= \text{freestream} \\
atm &= \text{atmospheric conditions} \\
CFD &= \text{truly evaluated through computational fluid dynamics} \\
exit &= \text{nozzle exit} \\
fuel &= \text{remaining hydrogen from combustion chamber} \\
GA &= \text{predicted by genetic algorithms} \\
i &= \text{i}th population \\
inlet &= \text{nozzle inlet} \\
j &= \text{fuel jet} \\
max &= \text{maximum value} \\
min &= \text{minimum value} \\
\end{align*} \]

Table 6

<table>
<thead>
<tr>
<th>( C_{H_2O,exit} ) (React On)</th>
<th>( C_{H_2O,exit} ) (React Off)</th>
<th>( C_{H_2,exit} ) (React On)</th>
<th>( C_{H_2,exit} ) (React Off)</th>
<th>( H_2 ) burned</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0153</td>
<td>0.00271</td>
<td>0.307</td>
<td>0.338</td>
<td>9.2%</td>
</tr>
</tbody>
</table>

Fig. 25. Mass fraction distributions at the nozzle outlet for the optimum injection configuration. (a) \( H_2 \) and (b) \( H_2O \).

Subscript

\( \infty \) = freestream  \\
atm = atmospheric conditions  \\
CFD = truly evaluated through computational fluid dynamics  \\
exit = nozzle exit  \\
fuel = remaining hydrogen from combustion chamber  \\
GA = predicted by genetic algorithms  \\
i = i\text{th population}  \\
inlet = nozzle inlet  \\
j = fuel jet  \\
max = maximum value  \\
min = minimum value  \\

References


