Physical Insight into Scramjet Inlet Behavior via Multi-Objective Design Optimization

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Scramjet propulsion is a promising technology for reliable and economical access to space and high-speed atmospheric transport. The inlet plays a key role in determining the performance of scramjets, in particular for the axisymmetric class of scramjet engines that are currently explored due to their advantages in numerous aspects. In the present study, a multi-objective design optimization based on evolutionary algorithms has been conducted with respect to four major inlet design criteria, that is, compression efficiency, drag, adverse pressure gradient, and exit temperature, where the former three criteria are used as the objective functions and the last one is the constraint function. The flowfields have been examined for representative geometries, and sensitivity analysis has been performed with the aid of surrogate modeling, revealing the major impact of the inlet exit radius, advantages of Busemann-type geometries in various aspects, and direct correlation of the inlet drag, exit temperature, and surface-heat transfer. The insight gained here can be usefully applied to the design of high-performance scramjet inlets.

Nomenclature

\[ A = \text{cross-sectional area, m}^2 \]
\[ dp/ds = \text{surface pressure gradient, Pa/m} \]
\[ F = \text{point among feasible individuals} \]
\[ h = \text{static enthalpy, J/kg} \]
\[ h_t = \text{stagnation enthalpy, J/kg} \]
\[ l_i = \text{length of } i\text{th ramp, m} \]
\[ m = \text{mass flow, kg/s} \]
\[ O = \text{point on Pareto optimal front} \]
\[ P = \text{static pressure, Pa} \]
\[ R = \text{specific gas constant, J/kg} \cdot \text{K} \]
\[ r = \text{radial coordinate or radius, m} \]
\[ S = \text{first-order sensitivity index} \]
\[ S_T = \text{total sensitivity index} \]
\[ s = \text{coordinate along inlet surface, m} \]
\[ T = \text{static temperature, K} \]
\[ x = \text{streamwise coordinate, m} \]
\[ \gamma = \text{specific heat ratio} \]
\[ \Delta Q = \text{heat transfer across inlet surface, J/s} \]
\[ \Delta \theta_i = \text{angle increment of } i\text{th ramp, deg} \]
\[ \eta_B = \text{compression efficiency} \]
\[ \theta = \text{wall inclination, deg} \]
\[ \theta_0 = \text{first Mach wave angle of full Busemann inlet, deg} \]
\[ \theta_i = \text{angle of } i\text{th ramp, deg} \]
\[ \tau = \text{wall shear stress, N/m}^2 \]
\[ \chi = \text{stream-thrust-average value of parameter } \chi \]

Subscripts

\[ c = \text{property of combustor} \]
\[ \text{isen} = \text{isentropic value} \]
\[ \text{max} = \text{maximum value} \]
\[ \text{pres} = \text{pressure component} \]

rel. = relative value
\[ t = \text{property of leading-edge tip} \]
\[ \text{visc} = \text{viscous component} \]
\[ 1 = \text{value at inlet entrance} \]
\[ 2 = \text{value at inlet exit} \]
\[ \infty = \text{freestream value} \]

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I. Introduction

Hypersonnic airbreathing propulsion offers the potential for reliable and economical transport for access to space and high-speed atmospheric cruise. In particular, scramjets (supersonic combustion ramjets) are a promising technology that can enable efficient and flexible transport systems, having marked significant milestones in the last decade: the world’s first supersonic combustion in the Hyper-X program in July 2002 [1,2], the fastest atmospheric flights recorded by NASA’s X-43A scramjet-powered vehicles in the Hyper-X program at Mach 6.8 (March 2004) and 9.6 (November 2004) [3], and the recent flight by the Boeing X-51A WaveRider, which recorded the longest scramjet burn duration of 140 s in May 2010 [4].

An axisymmetric scramjet configuration (Fig. 1) is currently being explored in ground and flight tests in Australia, following excellent performance demonstrated in shock tunnel testing [5,6]. Scramjet engines typically operate in a sequential process (Fig. 2): supersonic inflow is captured and compressed through the inlet to the desired high pressure and temperature. Fuel is injected and mixed with air, and combustion takes place in the downstream chamber. The reacted gas is expanded by the nozzle to produce thrust. Combined with innovative concepts, including inlet fuel injection and radical-farming shock-induced combustion, the simple axisymmetric configuration can bring about numerous advantages over complex three-dimensional geometries in aerodynamic and combustion efficiency, aero thermal and structural management as well as manufacture [2,8].

Axisymmetric scramjet inlets with high internal compression, however, are inherently difficult to start spontaneously during flight and are highly susceptible to unstart. Numerical investigations have recently been conducted by the authors to address this issue and probe the underlying physics. Time-accurate computations have revealed a substantial influence of shock wave/boundary layer interactions on the inlet starting mechanism, with formation of separation at compression corners playing an essential role in the unstarting process [9]. Various techniques have been examined numerically to overcome the problem, where instantaneous diaphragm rupture (with bleed addition) and sliding doors (or
The insight gained is described in a summary section under the construction of the inlet geometry, which is described in a summary section under. Despite the effectiveness of these methods, it is of crucial importance to design inlets that are naturally less susceptible to undesirable unstarting events for reliable and stable inflight operation of axisymmetric scramjets.

Design criteria for high-performance scramjet inlets typically include efficient compression with minimum viscous/shock losses, minimum contribution to the vehicle drag, and minimum adverse pressure gradient to prevent flow separation, while achieving adequate compression to induce ignition [11,12]. Such a design problem with multiple objectives would represent a formidable challenge for conventional gradient-based optimization approaches due to substantial complexity and nonlinearity introduced by highly coupled aerodynamic phenomena, including shock-shock interactions and shock wave/boundary layer interactions. Global search based on evolutionary algorithms is particularly suitable for the design of such complex systems that are characterized by nonlinear and nonsmooth design space being robust against the presence of local optima [13]. The use of population-based search in aerospace design, however, would commonly entail prohibitive computational cost due to a large number of function evaluations [e.g., computational fluid dynamics (CFD), finite-element analysis] required in the course of heuristic search.

Surrogate modeling can efficiently mitigate the computational load by replacing expensive function evaluations with approximation from meta-analysis models [14,15]. The capability and robustness of surrogate-assisted evolutionary algorithms (SAEAs) in multi-objective optimization have been demonstrated for various test problems featuring multiple objectives and constraints and numerous decision variables, accurately capturing complex nondominated optimal fronts with minimum function evaluations [16,17]. SAEAs have been applied to design optimization problems for various aerospace applications, including turbomachinery, transonic aircraft, rocket engine components, and hypersonic nose cones [15,18–21]. More recently, an advanced SAEA-based multi-objective optimization capability coupled with state-of-the-art CFD codes has been applied to design problems of various components and full flowpath of axisymmetric scramjet engines [22–25].

In the present research, a multi-objective design optimization with three objective functions and one constraint function has been performed for axisymmetric scramjet inlets to gain physical insight into the inlet flowfields. The optimization results and representative flowfields have been examined to investigate the key factors and underlying flow physics that can exert decisive influence on the inlet performance. The insight gained is described in a summary section intended to directly benefit the scramjet designer.

II. Approaches

1. Inlet Geometry

The inlet to be optimized in this paper comprises three ramps, as schematically shown in Fig. 3. The internal geometry is represented by eight parameters: the leading-edge nose-tip radius $r_1$, ramp lengths $l_1, l_2, l_3$, first ramp angle $\theta_1$, ramp angle increments $\Delta \theta_2, \Delta \theta_3$, and exit radius $r_e$ (or combustor radius). The baseline geometry is defined by the following values: $\theta_1 = 5.0 \, \text{deg}$, $\Delta \theta_2 = 5.3 \, \text{deg}$, $\Delta \theta_3 = 3.3 \, \text{deg}$, $l_1 = 0.204 \, \text{m}$, $l_2 = 0.069 \, \text{m}$, $l_3 = 0.039 \, \text{m}$, and $r_e = 0.035 \, \text{m}$. The inlet radius, which is measured from the axis of symmetry to the junction of the circular cross-sectional leading edge and the first inlet ramp, is fixed at 0.075 m to ensure constant mass flow entry, which effectively makes one of the ramp parameters dependent on the others for a given value of the combustor radius $r_e$ ($l_1$ is rendered to be such a dependent variable in this study). Also fixed is the leading-edge nose-tip radius $r_1 = 0.5 \, \text{mm}$ to focus on the influence of ramp geometries by freezing the entropy-layer effect originating from the leading edge. These assumptions, in effect, leave six parameters ($l_1, l_2, \theta_1, \Delta \theta_2, \Delta \theta_3$, and $r_e$) as design variables, or decision variables for optimization.

2. Flow Conditions

The freestream conditions are $M_{\infty} = 8.0$, $\rho_{\infty} = 1197 \, \text{Pa}$, and $T_{\infty} = 226.5 \, \text{K}$, assuming scramjet operation on a constant dynamic pressure trajectory of 53.6 kPa, at an altitude of 30 km. The rate of the mass flow captured by the constant inlet area is 0.78 kg/s. The Reynolds number based on the inlet radius (0.075 m) is $Re_{\infty} = 2.26 \times 10^6$.

B. Computational Fluid Dynamics

1. Computational Methods

Inlet flowfields are computed by using the state-of-the-art commercial solver CFD++, which has been employed by the Australian hypersonics network for scramjet research due to its demonstrated fidelity in hypersonic aerodynamics and aero thermodynamics [8,10,22–25]. An implicit algorithm with second-order spatial accuracy is used to solve the Navier–Stokes equations for steady flowfields, and convergence is accelerated by the multigrid technique. Standard air in thermochemical equilibrium state is assumed for the gas, and the inlet surface is assumed to be an isothermal cold wall at 300 K. The inflow is assumed to be fully turbulent and modeled by the two-equation shear-stress transport $k-\omega$ Reynolds-averaged Navier–Stokes model due to its demonstrated fidelity in the presence of adverse pressure gradient [28]. Computations are performed until the energy residual is reduced to the order of $10^{-5}$, based on a convergence study, where all objective and constraint functions have been found to vary less than 0.07% at higher orders, as seen in Fig. 4, which displays the variations of the deviation of the objective and constraint functions from their converged values (at an energy residual order of $10^{-6}$) for the baseline flowfield represented by a coarse mesh.

2. Computational Mesh

Two-dimensional structured meshes are generated by Glyph scripting within the commercial grid generator Pointwise [29] for the inlet geometry defined by the design parameters, as described in Sec. II.A.1 [Meshes are freshly generated from scratch for every individual to be evaluated, according to the design parameter...
A population-based evolutionary algorithm, in particular, the elitist nondominated sorting genetic algorithm (NSGA-II) [30], is employed for multi-objective design optimization [16,17]. Optimization occurs over 50 generations with 64 individuals in the population pool. A simulated binary crossover and polynomial mutation are used as recombination operators at a given probability (1.0 and 0.1, respectively) with a specified distribution index (10 and 20, respectively). The optimization process is efficiently assisted by replacing CFD evaluations for the objective and constraint functions with the estimations from various surrogate models, including response surface models, kriging approximations, and radial basis functions. Among the predictions from all these models, the one with the least error within a threshold of 10% is adopted to estimate the objective and constraint functions in lieu of actual CFD evaluation, and all individuals that are estimated to be superior to (not dominated by) the present optimums are verified by true CFD evaluation. All members in the population pool are truly evaluated by CFD every five generations, when the surrogate models are trained by using 90% of the solutions from the archive of all truly CFD-evaluated solutions. Variance-based global sensitivity analysis is performed to investigate the influence of the decision variables on the objective and constraint functions, based on the surrogate model with the best prediction accuracy as at the final generation. Evaluation is made for 10,000 sample data points represented by Sobol quasirandom numbers within the decision variable ranges [31,32].

2. Optimization Problem

Three characteristic parameters are chosen and used as objective functions to achieve the optimization goal to fulfill the inlet design criteria. The inlet performance is assessed by the compression efficiency, which is defined as the ratio of the enthalpy increase in ideal isentropic compression to that in the actual process, i.e. $\eta_B \equiv \frac{h(p_2, s_1) - h_1}{h_2 - h_1}$. This parameter is able to account for both shock and viscous losses less sensitively to nonuniform exit flows and inlet heat transfer, as compared with other efficiency parameters [11,12,33]. The inlet drag is evaluated as the second objective function to be minimized. The third objective is to minimize the greatest local adverse pressure gradient on the surface, which if too large would be responsible for boundary-layer separation and unstart (this quantity is treated as an objective function rather than a constraint in the present study because the maximum local adverse pressure gradient that the boundary layer can tolerate without separation has been found to largely depend on the state of the boundary layer [9,34]). A constraint function, a measure of the solution feasibility, is imposed on the mean temperature at the inlet exit (combustor entrance); this is required to be greater than 850 K so that local shock-induced hot structures further downstream can result in autoignition through the effect referred to as radical farming [8,35] because little is known on what constitutes desirable flow profiles at the combustor entrance [11]. The compression efficiency and mean temperature are evaluated by using stream-thrust averaged values [36]. The optimization problem can thus be stated as follows:

(continued)
CFD computation post-processing
optimization algorithms

Fig. 7 Optimization loop.

minimize: (1) \( 1 - \eta_B \)
(2) Drag
(3) \( dp/ds_{max} \)
subject to: \( T_2 \geq 850 \text{ K} \)

The ranges of the six decision variables (\( l_2, l_3, \theta_1, \Delta \theta_2, \Delta \theta_3, \) and \( r_c \)) have been set to allow reasonable geometric variations that are sufficiently large to investigate the influence of the decision variables on the inlet characteristics. The upper and lower bounds are shown later at the top and bottom, respectively, of the columns in Fig. 8.

III. Results

A. Optimization Results

1. Pareto-Optimal Front

The optimization has been performed until converged solutions have been achieved, where further evolution has led to little variation in the optimal solutions in the population pool. Figure 9 shows all the individuals (3329 solutions in total) that have been evaluated by CFD in the optimal solutions in the population pool. Figure 10 shows all the feasible and infeasible solutions according to the exit temperature criterion is found to lie at an approximate drag value of 157 N (Figs. 10b and 10c). Detailed analysis of the behavior follows later in the paper.

To investigate the results from the optimization, a few points have been selected that are different in one objective function but similar in the other two, thus facilitating comparison. The values of the objective and constraint functions are shown in Table 2 for such representative cases, and the points are displayed in Fig. 11 along with the 64 nondominated individuals that constitute the Pareto-optimal front (Representative points \( F_1, F_2 \) and \( F_3 \) are chosen from suboptimal individuals here, contrary to general approaches taken in multi-objective design optimization studies, where comparison is commonly made between multiple optimal points on the Pareto front. This is purposely done in the present research to highlight the effects of geometric features on the resultant inlet performance and facilitate the extraction of useful physical insight into inlet flowfields that is applicable to the design of generic inlets, not limited to the specific conditions and configurations considered in this paper).

The individual \( O_1 \) on the Pareto-optimal front has been nominated as the representative optimum owing to its balanced performance in all objective functions (The deviation of \( O_1 \) from the minimum of each objective function is 3.5%, 1.1%, and 28.5% for \( 1 - \eta_B \), drag, and \( dp/ds_{max} \), respectively, with higher priority given to the compression efficiency and drag rather than the adverse pressure gradient). The feasible individual \( F_1 \) offers a similar performance in all aspects, except the compression efficiency \( \eta_B \). The individual \( F_2 \) has similar characteristics to \( F_1 \), apart from a greater drag and exit temperature \( T_2 \). The individual \( F_3 \), on the other hand, features a performance comparable to \( O_1 \), except for a far greater maximum adverse pressure gradient \( dp/ds_{max} \). It is notable that the exit temperature \( T_2 \) of \( O_1 \) and \( F_3 \), which yield lower drag, lies at the verge of the feasibility limit (850 K), whereas that of the baseline geometry does not satisfy the feasibility criterion due to a low exit temperature.

2. Geometries

The geometries of the representative individuals are compared in Fig. 12 (not to scale). Plotted in Fig. 8 are the corresponding values of the objective and constraint functions and decision variables, along with the bounding values at the top and bottom of the columns. Considerable variations can be observed in the ramp angle \( \theta_1 \) and increments \( \Delta \theta_{2,3} \) as well as the exit radius \( r_c \) among the
representative solutions, whereas the levels of the ramp lengths $l_{2,3}$ are relatively similar. The total inlet length varies accordingly in response to the variation of the ramp angles. The effects of the geometric characteristics on the performance criteria are discussed in detail in the following sections.

**B. Observations**

1. **Compression Efficiency**

   The definition of the compression efficiency $\eta_B$ can be rewritten as $\eta_B = [h(T_2) - h(T_1)]/[h(T_2) - h(T_1)]$, where $T_2 = (p_2/p_1)^{(γ−1)/γ}/T_1$ according to the isentropic relations. For a given set of $T_1$ and $T_2$, by this definition, the compression efficiency $\eta_B$ becomes larger for greater $p_2/p_1$. The maximum efficiency of 100% is attained when the compression process is isentropic because the flow can hold the maximum possible static pressure at the exit in the absence of any total pressure loss. This parameter thus indicates how efficiently compression has been achieved for a certain temperature rise between the inlet entrance and exit.

   Figure 13 compares the compression efficiency for the representative geometries. $O_1$ and $F_3$ are found to yield higher compression efficiency in comparison with $F_1$ and $F_2$ and the baseline.

**Table 2** Objective and constraint function values of the representative points

<table>
<thead>
<tr>
<th>$1 - \eta_B$</th>
<th>Drag, N</th>
<th>$\frac{\Delta p}{\Delta s}$, Pa/m</th>
<th>$T_2$, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_1$</td>
<td>0.203</td>
<td>158</td>
<td>$8.95 \times 10^5$</td>
</tr>
<tr>
<td>$F_1$</td>
<td>0.298</td>
<td>167</td>
<td>$1.27 \times 10^6$</td>
</tr>
<tr>
<td>$F_2$</td>
<td>0.305</td>
<td>249</td>
<td>$1.48 \times 10^6$</td>
</tr>
<tr>
<td>$F_3$</td>
<td>0.204</td>
<td>159</td>
<td>$5.80 \times 10^5$</td>
</tr>
<tr>
<td>Baseline</td>
<td>0.301</td>
<td>124</td>
<td>$7.13 \times 10^5$</td>
</tr>
</tbody>
</table>

**Figure 10** Population projections on objective function planes.

**Figure 11** Pareto-optimal front and representative points.
that the average static pressure $\bar{p}_2$ achieved by $F_1$ is considerably lower than that achieved by $O_1$ and $F_2$, although similar compression levels have been achieved in terms of the static temperature $T_2$ (850–880 K) for these three cases. The static temperature and pressure profiles at the inlet exit plotted in Fig. 16 show considerably lower values for $F_1$, compared with those for $O_1$ and $F_2$, accordingly.

The flowfields for the $O_1$ and $F_1$ geometries are compared in Fig. 14 with respect to the Mach number and temperature. The $O_1$ flowfield is characterized by the impingement of the first reflected shock on the expansion corner at the junction of the inlet and combustor, which effectively allows the majority of flow compression to complete within the inlet, whereas the reflected shock wave impinges on the second inlet ramp, and flow compression occurs even in the combustor section in the case of the $F_1$ flowfield. According to the inviscid theory for axisymmetric flowfields, the so-called full Busemann inlet [37] is able to compress the airflow with a minimum entropy rise in case the inner inlet wall is represented by a fully concave surface as in the present study. Compression is achieved by means of isentropic compression through a series of Mach waves and a sole entropy jump across a terminating conical shock wave, as schematically shown in Fig. 17. It can be noted that the $O_1$ geometry is akin to the full Busemann inlet despite the presence of viscous effects and compression via a finite number of ramps, being able to fulfill the compression requirement of $T_2 \geq 850$ K with a minimum compression efficiency loss, whereas the shock waves originating from the second and third compression corners play a secondary role with a minor influence on the flow compression process.

2. Drag

The inlet drag is compared in Fig. 18, including the breakdown of the pressure (inviscid) and viscous contributions, which can be obtained by integrating the pressure and shear stress along the surface as $\text{Drag} = D_{\text{pres}} + D_{\text{visc}} = \int (p \sin \theta + \tau \cos \theta) 2\pi r \cos \theta$ ds. The distributions of $p \sin \theta$ and $\tau \cos \theta$ along the inlet surface are plotted in Fig. 19.

The higher drag level incurred by $O_1$, compared with the baseline, can primarily be attributed to a smaller exit radius $r_c$, i.e., an increased projected wall area on which the pressure can act in the $x$ direction. A somewhat higher drag is incurred by $F_1$, where the viscous contribution is augmented by a larger skin friction drag on the extended inlet surface, as seen in Figs. 14 and 19b. Nearly the same level of pressure drag, on the other hand, is incurred by $O_1$ and $F_1$ despite a larger frontal area of $O_1$, which is attributed to the higher pressure acting on a large extent of the second and third ramp due to the impingement of the main shock reflected on the symmetry axis in the $F_1$ flowfield (Figs. 14 and 19a). The greatest drag is experienced by $F_2$ due to its smallest exit radius $r_c$ and hence the largest frontal area, whereas the viscous drag is comparable to $F_1$ due to a similar inlet length (Fig. 19b).
3. **Pressure Gradient**

An adverse pressure gradient is a primary factor that is responsible for incipient flow separation and thus is minimized as one of the objective functions rather than a constraint because the degree of gradient that the boundary layer can withstand without separation is dependent on various factors such as the flow viscosity, turbulent kinetic energy, and thermal treatment of the wall surface [9, 34]. It has been found in Fig. 11 that \( F_3 \) is subject to a substantially higher degree of adverse pressure gradient, compared with the \( O_1 \) geometry, whereas the compression efficiency and drag levels are virtually the same for both.

The flowfields are compared in Fig. 20, where both geometries appear to be very similar. However, the wall pressure distributions plotted in Fig. 21 show a perpendicular rise immediately upstream of the inlet end in the \( F_3 \) case, which has led to an acute adverse gradient. This can be attributed to a smaller increment of the second ramp \( \Delta \theta_r \), making \( F_3 \) more susceptible to separation. The shock structure of the full Busemann inlet (Fig. 17) is advantageous not only for the compression efficiency but also in this aspect, preventing the shock impingement of the reflected shock upon the inner inlet wall, although particular care ought to be taken under off-design conditions. The highly localized, abrupt behavior of the pressure gradient observed here highlights the particular effectiveness of population-based global search, in contrast to gradient-based local search, for which the detection of the global optimum would be formidable if search were started with an unfavorable initial point in the nonsmooth design space.

4. **Temperature at Exit**

The flow temperature at the inlet exit plays a key role in the ignition process in the combustion chamber, particularly for radical-farming shock-induced combustion, where a sufficient amount of heat release is required to produce radicals and induce combustion [7, 8]. The stream-thrust average of the exit temperature is thus employed as a constraint function to judge the feasibility of the individuals. The

![Fig. 15 Temperature and pressure at inlet exit.](image)

![Fig. 16 Flow profiles at the inlet exit.](image)

![Fig. 17 Full Busemann inlet [38].](image)

![Fig. 18 Drag comparison and breakdown.](image)

![Fig. 19 Comparison of pressure and shear stress contributions to drag.](image)
temperature profiles at the inlet exit are compared in Fig. 15, and the temperature distributions are displayed in Fig. 22.

The feasible solutions are found to have satisfied the temperature requirement for self-ignition ($T_2 \geq 850$ K) in various ways; $F_1$ has cleared the minimum temperature criterion with a relatively large exit radius $r_c$ owing to the near-wall high temperature zone as seen in the thermal boundary layer in Figs. 16a and 22a. $F_2$, on the other hand, has attained a high temperature of $T_2 = 1108$ K, well over the self-ignition temperature, with high compression with a smaller radius close to the lower bound of the $r_c$ range, which has led to the emergence of an additional localized high-temperature region near the axis of symmetry (Fig. 22b), although it makes insignificant contribution to the stream-thrust-average temperature due to its proximity to the axis. $O_1$ and $F_3$ lie in the middle, the latter of which has a narrower and higher peak due to a slight modification of the shock structure caused by a smaller value of $\Delta \beta$.

It has been noted in the projection plots of the final population and individuals (Figs. 10b and 10c) that the feasibility of the solutions distinctly changes across a certain level of drag (157 N), suggesting the existence of a correlation between the drag and exit temperature. An analytical study has been conducted to elucidate the trend observed here: the inlet drag can be expressed as the difference in the stream thrust (momentum) between the incoming and outgoing airflows, i.e., $\text{Drag} = F_{\text{in}} - F_{\text{out}}$, according to the momentum balance theory. Coupling the definitions of the mass flow $\dot{m} = \dot{\rho} U A$ and...
where \( \Delta Q \) is the heat transfer across the inlet surface, and the static enthalpy \( \tilde{h}_2 \) is a function of the exit temperature \( T_2 \) for the thermally equilibrium air assumed in this study.

Figure 23 compares the CFD solutions evaluated during the optimization with theoretical results. The dashed line is the analytical curve obtained from Eq. (1) by assuming no heat transfer (\( \Delta Q = 0 \)) for a given inflow \( (F_{\text{in}} \equiv 1901 \text{ N}, \dot{m} = 0.786 \text{ kg/s}, \text{ and } h_1 = 3.15 \text{ MJ/kg}) \), whereas the squares denote the solutions obtained by substituting the heat transfer values calculated numerically in CFD into \( \Delta Q \) in Eq. (1). The close agreement between the squares and CFD solutions with an rms error of 0.32% reassures that the deviation of the CFD results from the dashed analytical curve solely originates from the surface-heat transfer, which accounts for 15.7% drag increase on average, compared with the drag in the absence of heat transfer (in other words, the inlet drag can be expressed as a function of the exit temperature only in the case of adiabatic inlet walls).

5. Sensitivity Analysis

Global sensitivity analysis has been performed, based on the prediction from the best surrogate model with the least error that has been trained by using the archive of the CFD solutions evaluated in the course of the optimization [15,31]. The first-order sensitivity index \( S \) represents the main effect of the input parameter (decision variable) on the output parameter (objective/constraint function). The total sensitivity index \( S_T \) is the sum of all the effects, including first-order as well as higher-order ones, which account for the interactions with the other parameters. A small \( S_T \) value indicates that the parameter is noninfluential and can be fixed anywhere in its distribution without affecting the variance of the output [31].

The sensitivity indices \( S \) and \( S_T \) are plotted in Fig. 24 for the compression efficiency \( \eta_B \), whose indices are found to be very similar to those for the drag and exit temperature \( T_2 \) (No reasonable indices have been obtained for the maximum adverse pressure gradient \( dp/dx_{\text{max}} \) due possibly to the highly local nature of this parameter, as compared with the others, which are integral quantities). It can be seen that the radius at the inlet exit \( r_2 \) exerts dominant influence on these objective and constraint functions. The ramp angles and lengths have a rather minor influence, but the former can exert more impact in conjunction with other parameters, as indicated by their larger proportions in \( S_T \) than in \( S \).

This result is consistent with the observation that the compression efficiency can be maximized by geometries akin to the full Busemann inlet, where the following relation holds for the overall inlet length and other geometric parameters according to Fig. 17:

\[
l_1 + l_2 + l_3 = r_1 / \tan \theta_0 + r_2 / \tan \theta_3,
\]

where \( r_1 \) is constant at 0.075 m, and \( \theta_0 \) is determined by the inflow Mach number \( M_1 \) and the first ramp angle \( \theta_1 \) in the present study. The total inlet length is thus expressed as a function of \( r_2 \) and \( \theta_3 \), which is determined in relation to the exit Mach number \( M_2 \). The inlet exit radius \( r_2 \) can then be directly correlated with the exit temperature \( T_2 \) and surface-heat transfer \( \Delta Q \) due to the energy balance equation \( h_1 + \Delta Q / \dot{m} = \tilde{h}_2 + 1/2 \dot{m}^2 / 2 \dot{m}^2 / 2 \Delta Q / \dot{m} \). This correlation makes \( r_2 \), a key factor in determining the compression efficiency, inlet drag, and exit temperature as a consequence.

6. Summary

The insight gained into axisymmetric scramjet inlet flowfields as a result of the present study is summarized here:

1) The inlet exit radius (or contraction ratio) has the most dominant effect on the compression efficiency, inlet drag, and exit temperature among all design parameters. The ramp angles have secondary influence on these design criteria, which are little affected by the ramp length.
2) Both compression efficiency losses and abrupt adverse pressure gradient (hence boundary-layer separation) can be prevented effectively by employing Busemann-type flow structures characterized by initial isentropic compression followed by a terminating conical shock wave, which impinges on the junction of the inlet and combustor.

3) The inlet drag is directly linked with the exit temperature with secondary influence of heat transfer across the inlet surface (or no influence in case of adiabatic surface), indicating a monotonic increase of the inlet drag as a function of the exit temperature. This insight consequently provides the following remarks that can be applied to the design of axisymmetric inlets:

4) High inlet performance can be achieved by Busemann-type geometries with necessary modifications, including truncation, multiple ramps, and corrections, for boundary/shock layers, in accordance with the requirements.

5) The inlet drag can be reduced by mitigating the minimum temperature requirement at the inlet exit (or combustor entrance) and further reduction (up to 15.7% in the present study) can be achieved by suppressing surface-heat transfer.

IV. Conclusions

A multi-objective design optimization has been performed for axisymmetric scramjet inlets with respect to four design criteria, namely, the compression efficiency, drag and maximum adverse pressure gradient as objective functions, and the exit temperature as a constraint function. The inlet geometry comprising three compression ramps has been optimized by coupling surrogate-assisted evolutionary algorithms and a computational fluid dynamics solver for viscous equilibrium flowfields, resulting in a Pareto-optimal front. The flowfields have been scrutinized for the representative cases, and sensitivity analysis has been applied to the stored solution archive by using the trained surrogate models.

The inlet exit radius is found to be the most influential parameter on the compression efficiency, inlet drag, and exit temperature. Busemann-type geometries are advantageous in maximizing the compression efficiency while preventing abrupt adverse pressure gradient, which can potentially induce unfavorable flow separation. The inlet drag has been found to be determined primarily by the exit temperature and secondarily by the surface-heat transfer for inlets with fixed mass flow capture. These results suggest that the inlet drag can be designed in a decoupled manner for given combustor specifications and that the inlet drag need not be considered as an objective function to minimize if the minimum temperature is specified at the combustor entrance, simplifying the overall scramjet design process.

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References


[29] Pointwise, Software Package, Ver. 16.02, Pointwise, Inc., Fort Worth, TX, 2008.


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