Nozzle Design Optimization for Axisymmetric Scramjets by Using Surrogate-Assisted Evolutionary Algorithms

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Scramjet propulsion is a promising hypersonic airbreathing technology that offers the potential for efficient and flexible access to space and high-speed atmospheric transport. Robust nozzle design over a range of operating conditions is of critical importance for successful scramjet operation. In this paper, shape optimization has been performed with surrogate-assisted evolutionary algorithms to maximize the thrust generated by an axisymmetric scramjet nozzle configuration, including the base flow and external surface for cruise conditions at Mach 8 at two altitudes with and without fuel. The optimization results have been examined in a coupled numerical/analytical approach in order to identify the key design factors and investigate the effects of design parameters. It has been found that the optimum nozzle geometries are characterized by bell-type shapes for the fuel-on conditions, whereas the optima for the fuel-off case feature nearly conical shapes. Their robustness in thrust production has been demonstrated by cross-referencing the optimum geometries at off-design altitudes. The nozzle length and radius have been found to be the most influential parameters in all considered conditions, with their optimum values determined based on the balance between inviscid and viscous force components, whereas the other parameters have minor impact on the total axial force.

Nomenclature

- \( A \) = axial force coefficient
- \( C \) = conditional expected value
- \( F_{x} \) = total axial force, N
- \( f_{x} \) = axial force component, N
- \( H \) = altitude, km
- \( h \) = height, m
- \( l \) = length, m
- \( M \) = Mach number
- \( N_{val} \) = number of CFD evaluations
- \( p \) = static pressure, Pa
- \( q \) = dynamic pressure, Pa
- \( Re \) = Reynolds number
- \( r \) = radius m or radial coordinate, m
- \( S \) = entire surface of interest
- \( S_{0} \) = inlet airflow capture area, m²
- \( S_{f} \) = first-order sensitivity index of \( x_{i} \)
- \( S_{T} \) = total-effect sensitivity index of \( x_{i} \)
- \( s \) = coordinate along surface, m
- \( T \) = static temperature K or thrust, N
- \( V \) = variance
- \( X \) = input matrix for sensitivity analysis
- \( x \) = axial coordinate, m
- \( x_{i} \) = \( i \)th design (decision) variable
- \( y \) = output vector for sensitivity analysis
- \( \gamma \) = specific heat ratio
- \( \delta \) = vector direction, deg
- \( \theta \) = angle, deg
- \( \Delta \theta \) = angle increment, deg
- \( \lambda \) = vector magnitude m

- \( \sigma \) = standard deviation
- \( \tau \) = surface shear stress, N/m²
- \( \Phi \) = fuel/air equivalence ratio

Subscripts

- \( a \) = arc at the nozzle entrance
- \( b \) = back
- \( c \) = combustor
- \( f \) = forebody
- \( ite \) = inner trailing edge
- \( Li \) = variable index for the lower bound
- \( n \) = nozzle
- \( norm. \) = normalized
- \( ote \) = outer trailing edge
- \( t \) = leading-edge nose tip
- \( Ui \) = variable index for the upper bound
- \( \infty \) = freestream

I. Introduction

Hypersonic airbreathing propulsion offers great potential for reliable, reusable, and economical systems for access-to-space and high-speed atmospheric cruise for both civilian and strategic purposes. Scramjet (supersonic combustion ramjet) propulsion [1], in particular, is a promising technology that can enable efficient and flexible transport systems by removing the need to carry oxidizers and other limitations of conventional rocket engines. The last decade has seen remarkable milestones in scramjet development. Examples include the world’s first in-flight demonstration of supersonic combustion via scramjet technology, conducted by The University of Queensland (UQ) in the HyShot II Program [2,3]; the fastest flight record via airbreathing engines attained by NASA’s X-43A scramjet-powered vehicle in the Hyper-X program at Mach 9.6 in November 2004 [4]; and the longest scramjet burn duration of more than 200 s, achieved in a recent flight of the Boeing X-51A WaveRider in May 2010 [5].

The SCRAMSPACE project [6] is now underway as an international collaboration led by UQ, where a Busemann-like internal-compression axisymmetric scramjet (Fig. 1) is being explored, following the excellent performance demonstrated in shock tunnel testing [7]. The simple axisymmetric configuration, in conjunction with innovative concepts such as inlet fuel injection and radical-farming shock-induced combustion [8,9], offers advantages...
over complex three-dimensional geometries in various aspects, such as aerodynamic and combustion efficiency, thermal and structural management, as well as manufacturing. A scramjet engine typically consists of an inlet, combustor, and nozzle (Fig. 2) and operates in a key sequential process: Hypersonic airflow is captured and compressed through the inlet to the desired high pressure and temperature. Fuel is injected upstream of the combustor to be mixed with the air, and supersonic combustion takes place in the combustion chamber. The resultant reacted gas is expanded in the nozzle section to produce thrust. The scramjet nozzle thus plays a primary role in determining thrust production and, hence, overall engine performance, making it of crucial importance for successful scramjet operation to design high-performance nozzles for a range of operating conditions. Such robust nozzle design, however, would pose a significant challenge for conventional design optimization approaches due to the nonlinearity of the systems associated with highly coupled aerodynamic and aerothermal phenomena involved in scramjet flowfields such as shock–shock interactions, shock-wave/boundary-layer interactions, and finite-rate chemical reactions.

Evolutionary algorithms, by virtue of global search, are particularly suitable for such complex systems with nonsmooth design space [10] but would inherently entail prohibitive computational cost due to a large number of function evaluations required in population-based search, as is often the case with aerospace applications. Surrogate models can effectively mitigate the computational load by replacing expensive function evaluations with approximation from meta-models [15]. An advanced design methodology coupling SAEAs with high-fidelity computational fluid dynamics (CFD) solver has been developed at UQ in collaboration with University of New South Wales (UNSW). Extensive optimization studies have been conducted at UQ by applying this powerful design capability to optimization problems with single and multiple objectives for various components of axisymmetric scramjet engines [16–18] in line with the configuration to be employed in the SCRAMSPACE project.

This paper presents the results and physical insight obtained from an SAEAs-based, single-objective design optimization study performed to maximize the total thrust force of an axisymmetric nozzle configuration, including the base flow and rear part of the external wall at two altitudes on a constant Mach 8 trajectory, aiming at sustainable scramjet-powered cruise. The behavior of the optimum geometries has been investigated by examining the flowfields in various aspects, and the archive of CFD-evaluated solutions have been scrutinized by means of various data-mining techniques, including variance-based sensitivity analysis based on trained surrogate models in order to identify the key design factors and underlying physics.

II. Approaches

A. Conditions and Configurations

1. Nozzle Configuration

The axisymmetric scramjet engine considered in the present study is composed of an inlet with upstream fuel injection followed by the combustor and nozzle section based on the SCRAMSPACE configuration [6]. The inlet comprises three ramps, followed by a combustion chamber. This configuration features an external forebody and a flat base at the rear end due to technical requirements associated with separating the scramjet from its rocket booster. The full scramjet geometry is parameterized in Fig. 3, represented by 20 design parameters that define the angle, length, and curvature of the components. In the present optimization study, all geometric parameters upstream of the axial location of the nozzle entrance have been fixed (the radius at the nozzle entrance is constant at 0.0351 m, the same as that at the combustor exit). Only the downstream components are varied here, including the nozzle, back base, and rear part of the external surface, as highlighted in Fig. 3. The nozzle contour is composed of an initial arc represented by the radius \( r_e \) and \( \theta_e \) at the entrance. It is followed by a cubic Bézier curve defined by the vector magnitude \( L_n \) for the upstream end, where the direction is determined to ensure smooth junction with the initial circular arc, and the vector magnitude \( \lambda_{in} \) and direction \( \delta_{in} \) for the inner trailing edge, where \( \delta_{in} \) is defined as an angle relative to the straight line that connects both end points of the nozzle curve, as indicated by a dashed line in Fig. 3. The outer radius of the scramjet is constant in accordance with the upstream components fixed to the baseline geometry, which is relevant to the configuration to be explored in the SCRAMSPACE project [6]. (The specification is given below.) The position of the trailing edge is dictated by the nozzle length \( L_n \) and exit radius \( r_e \). The height of the rear base, which needs to be no less than 40 mm due to technical requirements, is determined in relation with \( r_e \) and the outer radius.

This parametric representation of the nozzle geometry is particularly adopted in this study, based on the report that parabolic bell nozzles composed of an initial arc and a quadratic Bézier curve are capable of approximating thrust-optimized contoured nozzles [19], and the (3rd-order) cubic Bézier curve adopted in the present configuration is able to represent the (2nd-order) quadratic curve [20]. To verify this, an inviscid shape optimization has been performed for the inner nozzle contour alone, defined by the present geometric representation, based on the procedure described later in this section, except that the nozzle length has been fixed at 0.385 m so as to allow comparison with analytical shapes. The chemically frozen state is assumed for the nozzle flowfields, and the nozzle inflow is assumed to be uniform flow obtained by stream-thrust averaging [21] the fuel-on profile at 27 km, plotted in Fig. 4 (with an average specific heat ratio \( \gamma \) of 1.27) in this preliminary optimization run. The optimum geometry reached after 100 generations is compared in Fig. 5 with the parametric approximation of the optimum bell nozzle contour derived theoretically by Rao [19], and the corresponding 15 deg half-angle conical nozzle contour with an expansion ratio of 15.4 and an initial circular arc with a radius of 0.382\( r_e \), where \( r_e \) is the combustor radius [22]. Almost indiscernible difference can be found between the optimum and Rao’s parabolic contours, except slight deviation near the trailing edge, both producing 754 N thrust whereas the conical nozzle yields 716 N. This result validates the geometric representation adopted in the present study.
2. Freestream Conditions

Cruise conditions at two altitudes, i.e., 27 and 32 km, are considered as design points for the nozzle optimization aiming at scramjet operation at Mach 8. The freestream conditions are tabulated in Table 1 for the two design points, where the Reynolds number is based on the inlet capture radius of 0.075 m.

B. Computational Fluid Dynamics

1. Flow Solver

Scramjet flowfields are computed here by using a commercial solver CFD++ [23], which is a high-fidelity code that has been subjected to extensive validation work and employed by the Australian hypersonics network for scramjet research because of its demonstrated fidelity in hypersonic aerodynamics and aerothermodynamics [9,24,25]. An implicit algorithm with second-order spatial accuracy is used to solve the Navier–Stokes equations for steady flowfields involving finite-rate chemical reactions, and convergence is accelerated by the multigrid technique. The gas composition and chemical reactions are represented by Evans and Schexnayder’s model [26], which consists of 25 elementary reactions among 12 species, including hydrogen-air combustion and nitrogen reactions. An advanced wall-function technique [27] is used for near-wall treatment. The inflow is assumed to be fully turbulent and modeled by the two-equation supersonic transport $k-\omega$ Reynolds-averaged Navier–Stokes model [28]. Computations are performed until 1000 iteration steps by restarting from a converged solution for the baseline geometry, with the Courant–Friedrichs–Lewy number ramped from 0.1 to 25 over the initial 100 steps or until the energy residual drops to the order of $10^{-5}$ (whichever event occurs first).

2. Computational Mesh

Two-dimensional structured meshes are generated by Glyph scripting in a commercial grid generator Pointwise [29] for the rear section of the axisymmetric scramjet represented by the seven geometric parameters described previously. The mesh comprises

\[ M \rightarrow \frac{1}{2} \int_{\Omega} \rho u^2 dV \]

\[ P \rightarrow \frac{1}{2} \int_{\Omega} \rho u^2 dV \]

\[ T \rightarrow \frac{1}{2} \int_{\Omega} \rho u^2 dV \]

\[ c_{H_2} \rightarrow \frac{1}{2} \int_{\Omega} \rho u^2 dV \]

\[ \text{Fig. 3 Parametric representation of the scramjet geometry.} \]

\[ \text{Fig. 4 Nozzle inflow profiles.} \]
35,850 cells (36,340 nodes) with the non-dimensional distance $y^+$ varying from 0.32 to 3.2 along the wall surface, as displayed in Fig. 6. This mesh resolution has been selected in order to strike the balance between the simulation fidelity of CFD and computational cost, based on a mesh sensitivity study that was performed for a previous optimization study with a similar nozzle configuration [17] (the mesh convergence will be revisited later in Sec. III.C.1, where the choice of this resolution is to be justified).

3. Flow and Boundary Conditions

The conditions imposed on the boundaries of the computational domain are schematically presented in Fig. 7. The body surface is assumed to be isothermal cold walls of 300 K, applicable to impulse facility or short duration flight testing. The inflow profiles imposed at the nozzle entrance are plotted in Fig. 4 for both design altitudes, i.e., 27 and 32 km, in the presence/absence of fuel, having been obtained from separate CFD simulations for the nominal SCRAMSPACE specification [6]. The profiles without fuel have been extracted from axisymmetric CFD runs with the baseline geometry. Those with fuel, on the other hand, have been obtained by circumferentially averaging the flow quantities sliced at the plane of the combustor exit from three-dimensional CFD runs in the presence of inlet fuel (hydrogen) injection through six portholes on the second inlet ramp at a fuel/air equivalence ratio of $\Phi = 0.5$. The freestream condition is also applied to the top boundary, whereas a supersonic outflow condition is imposed on the downstream boundary, which is positioned 0.15 m downstream of the rear base.

C. Optimization

1. Optimization Chain

Design optimization is performed in an iterative manner in an automated sequential process. The optimization chain is schematically presented in Fig. 8, consisting of four steps, namely, mesh generation (pre-processing), CFD computation (function evaluation), post-processing, and optimization algorithms. A computational mesh is generated at the first phase, based on the set of the design (decision) variable values provided by the optimization algorithms. The flowfield is evaluated by CFD (unless it is replaced by surrogate prediction), and the solution is passed on to the post-processor, which extracts information of interest, that is, the objective functions as well as the constraint functions (if applicable). The optimization algorithms assess and rank the individuals, based on the extracted information, in order to determine the population members for the next generation via evolutionary operations, i.e., selection, recombination, reproduction, and mutation. The decision variables of the new population are fed into the mesh generation to close the loop for the iterative process.

2. Optimization Algorithms

Population-based evolutionary algorithms developed at UNSW are employed as the design optimization algorithms [30,31]. In particular, use is made of an elitist real-coded genetic algorithm with simulated binary crossover and polynomial mutation [10,32]. Optimization is carried out with a population of 32 individuals. A simulated binary crossover and polynomial mutation are used as recombination operators at a given probability (1.0 and 0.1, respectively) with a specified distribution index (10 and 20, respectively). The optimization process is efficiently assisted by prediction from various surrogate models [11] including the response surface models [33], kriging approximations [34], and radial basis function network [35]. The surrogates are trained every five

<table>
<thead>
<tr>
<th>Altitude $H$, km</th>
<th>Velocity $u_{\infty}$, m/s</th>
<th>Static pressure $p_{\infty}$, Pa</th>
<th>Static temperature $T_{\infty}$, K</th>
<th>Dynamic pressure $q_{\infty}$, kPa</th>
<th>Reynolds number $Re_{\infty}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>2425</td>
<td>1847</td>
<td>223.7</td>
<td>82.5</td>
<td>$3.53 \times 10^5$</td>
</tr>
<tr>
<td>32</td>
<td>2398</td>
<td>868</td>
<td>228.7</td>
<td>38.8</td>
<td>$1.61 \times 10^5$</td>
</tr>
</tbody>
</table>
generations by using 90% of the solutions from the archive identified by using the \(k\)-means clustering algorithms [31], whereas the remaining 10% is used for validation, i.e., computation of the prediction error. The approximation given by the model with the best prediction accuracy is adopted to replace actual CFD evaluations when two conditions are satisfied, that is, a) the solid diagonal distance of the solution to the closest point in the archive is within a given threshold of 5%, and b) the root-mean-square (rms) error of the prediction is less than a threshold of 10% for the objective function (total axial force \(F_x\)).

3. Optimization Problem

The total axial force \(F_x\) (positive for drag and negative for thrust) is employed as the sole objective function to minimize, although no constraint function is employed in the present study. Seven design parameters are used as the decision variables with respective limits (the upper and lower bounds of the decision variables are displayed later in Fig. 9 at the top and bottom of the corresponding columns, respectively). The optimization problem is thus stated as:

\[
\text{minimize: total axial force } F_x
\]

\[
\text{decision variables: } x_{Li} \leq x_i \leq x_{Ui} \quad (i = 1, \ldots, 7)
\]

4. Sensitivity Analysis

Variance-based global sensitivity analysis is performed in order to assess the impact of each decision variable \(x_i\) (input) on the total axial force \(F_x\) (output). In particular, a numerical procedure based on Sobol’s variance decomposition [36] is employed to derive the sensitivity indices, based on surrogate prediction [12]. Input matrices \(X\) of a base sample number of 10,000, and seven columns for decision variables are built by using quasi-random numbers [37] within the range for each decision variable. Output vectors \(Y\) are obtained by feeding the input matrices into the surrogate model with the best prediction accuracy. The first-order indices \(S_i\)’s and total-effect indices \(S_{Ti}\)’s are calculated by the method outlined in Reference [38], defined as

\[
S_i = \frac{V[E(Y|X_i)]}{V(Y)}, \quad S_{Ti} = 1 - \frac{V[E(Y|X_{-i})]}{V(Y)}
\]

respectively, where \(V\) and \(E\) are the variance and conditional expected value, respectively.

III. Results

A. Design Optimization

1. Objective Functions

Evolutionary algorithm-based optimizations have been performed for each design condition. These studies have been carried out for up to 30 generations in each study. The total axial force and its inviscid and viscous components (defined later in Eq. (2) in Sec. III.B.2) are shown in Table 2 for the optimal individuals as of the final generation, along with the comparison of the axial force coefficient \(C_A\) \(= -F_x/q_0S_0\), where \(S_0\) is the inlet airflow capture area \(0.0177 \text{ m}^2\) between the optimum and baseline geometries, which indicates respectable increase in the nozzle thrust owing to design optimization in all cases and insensitivity of the coefficient to the

<table>
<thead>
<tr>
<th>Design condition</th>
<th>Max (-F_x) [N]</th>
<th>(C_A)</th>
<th>(1% \sigma(-F_x)) [N]</th>
<th>(N_{eval})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Inviscid Viscous Optimin Baseline</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27 km, fuel-on</td>
<td>574 665</td>
<td>-91.0</td>
<td>0.394 0.338</td>
<td>0.984 310</td>
</tr>
<tr>
<td>27 km, fuel-off</td>
<td>107 189</td>
<td>-82.1</td>
<td>0.074 0.021</td>
<td>0.245 655</td>
</tr>
<tr>
<td>32 km, fuel-on</td>
<td>247 293</td>
<td>-45.6</td>
<td>0.361 0.307</td>
<td>0.532 177</td>
</tr>
<tr>
<td>32 km, fuel-off</td>
<td>48.4 90.5</td>
<td>-42.1</td>
<td>0.071 0.022</td>
<td>0.165 354</td>
</tr>
</tbody>
</table>

Table 2 Axial force and number of true CFD evaluations as a result of optimization.
deviations of darker lines suggest the rather minor role played by the variables. It is notable that the superior individuals are commonly characterized by specific values of the nozzle length \( l_n \) and radius \( r_n \), whereas the other parameters are allowed to have perturbations to some extent to achieve the total axial force comparable to the optimum value, as indicated by larger bandwidths.

The nozzle contour profiles of all evaluated individuals are superimposed in Fig. 12, corresponding to those plotted in Fig. 11. The optimum geometries feature similar bell-type shapes with prominent surface curvature for both 27 and 32 km in the presence of fuel, where the radius of the initial contour arc \( r_n \) is larger for the former than the latter. On the other hand, the optima without fuel are characterized by near conical shapes with approximately straight contours and small initial arcs, as also seen in Fig. 13, which compares the optimum and baseline geometries (note that the axes are not to scale). The driving mechanism of these characteristics of the optimum geometries is to be scrutinized from physical perspectives in Sec. III.B.

3. Sensitivity Analysis

The impact of the design variables \( x_i \)'s on the objective function, namely the total axial force \( F_x \), has been examined by means of variance-based global sensitivity analysis by using surrogate models trained with truly CFD-evaluated solutions, as outlined in Sec. II.C.4. The first-order (\( S_i \)) and total-effect (\( S_{i*} \)) indices are plotted in Fig. 14 for the total axial force as well as its viscous and inviscid components for all design conditions. It is noticeable that the inviscid components are commonly characterized by the sum of the first-order sensitivity indices smaller than unity (\( \Sigma S_i < 1 \)), whereas that of the total-effect indices being greater than unity (\( \Sigma S_{i*} > 1 \)) in all design conditions. The effects of individual design variables \( x_i \)'s on the objective function \( F_x \) are linearly additive if the sums are equal to unity, i.e., \( \Sigma S_i = 1 \) and \( \Sigma S_{i*} = 1 \), and the difference between the total-effect index \( S_{i*} \) and the first-order index \( S_i \) is indicative of the degree of the involvement of the design variable \( x_i \) in interactions with other design variables [38].

It therefore suggests that design variables with greater values of \( S_{i*} \) than those of \( S_i \), such as the nozzle length \( l_n \), nozzle arc angle \( \theta_n \), and inner trailing-edge vector direction \( \delta_{nu} \), are more actively involved in the interactions than other parameters. In particular, the first-order \( S_i \) is seen to be barely discernible for the latter variables (\( \theta_n \) and \( \delta_{nu} \)), indicating that these parameters cannot exert impact on \( F_x \) by themselves but can introduce some influence only in cooperation with other variables. It can also be seen that the deviations of the sum of the sensitivity indices from unity (\( 1 - \Sigma S_{i*} \) and \( \Sigma S_{i*} - 1 \)) are greater in the fuel-off case than in the fuel-on for both altitudes, indicating the presence of more interactions in the fuel-off conditions.

The viscous components, on the other hand, exhibit the sum very close to unity for both sensitivity indices (\( \Sigma S_i \approx 1 \) and \( \Sigma S_{i*} \approx 1 \)), indicative of additive models in which total effects on \( F_x \) is the summation of the individual effects of these influential variables, i.e., the nozzle length \( l_n \) and radius \( r_n \).

4. Force Contour Plots

The total axial force \( F_x \) and its components have been found to be the most sensitive to the nozzle length \( l_n \) and radius \( r_n \) in the sensitivity analysis, whereas the other variables have rather minor impact. To investigate the roles played by these primary variables, the variations of \( F_x \) are presented in the form of contour plots with particular focus on \( l_n \) and \( r_n \) in Figs. 15 and 16 for the altitude of 27 and 32 km, respectively, where the contours have been produced by applying a surface-fitting method\(^{11}\) to surrogate predictions. The circles represent all the individuals truly evaluated by CFD, and the

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\(^{11}\)Fitting-surfaces have been generated by means of local regression using robust weighted linear least squares (Loess) with a quadratic model and the bi-square weight method, where the regression weight functions have been determined, based on surrogate predictions (including not only \( l_n \) and \( r_n \) but also all other design variables) for data points within a 25% span among 10,000 quasi-random points in the design space.

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Fig. 10 Number of true CFD evaluations and progression of total axial force (27 km, fuel-on).

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2. Design Variables

The progression of these parameters over generations is presented in Fig. 11 in the form of the parallel coordinated plots [39] for all individuals that have been truly evaluated by CFD. The normalized values for each parameter are connected by lines whose darkness represents the generation (darker lines correspond to later generations). The values of the optimum individuals are shown by the thickest line with circular markers, whereas black lines stand for the other elites whose total axial force \( F_x \) has saturated at the prescribed upper bound (\( F_x \leq 600 \) N). It is evident that the nozzle radius \( r_n \) is more actively involved in the optimization of all design conditions, suggesting the possibility of further gain in case the restrictions were loosened.
color inside the circles indicate the actual function values from CFD (the color mismatch between inside and outside of the circles can be attributed to the prediction errors in surrogate modeling and smoothing in surface fitting).

The contours for the fuel-on conditions plotted in Fig. 15a mainly consist of lines in the lateral (horizontal) direction, indicating the primary dependency of the total axial force $F_x$ on the nozzle radius $r_n$ rather than the nozzle length $l_n$. The fuel-off case (Fig. 15b),
on the other hand, exhibits the presence of transverse lines, which suggests a higher degree of the dependency of \( F_s \) on the nozzle length \( l_n \) than that in the fuel-on case in accordance with the observations in the sensitivity analysis in Sec. III.A.3. It is noteworthy that a larger number of contours are crossed by varying the nozzle length \( l_n \) while maintaining the nozzle radius \( r_n \) in the vicinity of the optimum geometry, in the fuel-off case than in the fuel-on case, indicating higher robustness of the fuel-on optimum design against perturbation of the nozzle length. The same trends are observed in the contour plots for the altitude of 32 km (Fig. 16).

The contours of the inviscid and viscous axial force components are plotted in Figs. 17 and 18 for the fuel-on and fuel-off case at 27 km, respectively, in order to examine their contributions to the total axial force. It can be seen in both cases that the inviscid force increases as the length \( l_n \) and radius \( r_n \) of the nozzle increase on the whole. However, horizontal contours near the lower \( r_n \) limit (at the bottom of the figures) indicate that the inviscid force of low-expansion nozzles is insensitive to the length \( l_n \), whereas vertical contours toward the lower \( l_n \) limit and upper \( r_n \) range (at the top left of the figures) indicate that the inviscid force of short nozzles becomes insensitive to the radius \( r_n \) or expansion ratio when \( r_n \) is large. The former trend can be attributed to the relatively small variation of the wall inclination angle \( \theta \) [in Eq. (2)] caused by the variation of the length for small nozzle radii, and the latter can be due to significantly low surface pressure as a result of overexpansion in short nozzles with large exit radii, which is unable to make responsible contribution to the nozzle thrust any more.

The viscous drag decreases almost linearly in proportion to the nozzle length while the nozzle radius has a secondary effect, slightly mitigating the viscous drag as it increases. The tendency observed in the total axial force in Fig. 15 is the superimposition of the trends of the inviscid and viscous forces described here. It is noteworthy that the inviscid force varies more radically when the nozzle length is varied with the radius fixed in the fuel-off case than in the fuel-on case, indicating higher sensitivity of the fuel-off case to the nozzle length, in agreement with the characteristics represented by the sensitivity indices displayed in Fig. 14a.

### B. Analysis

#### 1. Flow Structure

The flowfields for the baseline and optimum geometries are visualized\(^{11}\) with respect to the Mach number and compared for the fuel-on and fuel-off cases at the two design altitudes in Figs. 19 and 20. It is seen that the optimized contours allow the nozzle inflow to expand more gradually, and the exit flow has greater axial velocity components, as compared to the baseline cases at both altitudes. Subsequently, a larger ratio between the axial and radial (non-axial)

#### 2. Surface Forces

The axial force acting on axisymmetric surface can be obtained by integrating the pressure and shear stress as follows:

\[
F_s = F_{s_{\text{viscous}}} + F_{s_{\text{viscous}}} = \int (p \sin \theta) 2\pi r \, ds + \int (-\tau \cos \theta) 2\pi r \, ds = \int (p \sin \theta - \tau \cos \theta) 2\pi r \, ds
\]

\(^{11}\)Flowfields visualized in this section have been re-computed with the fine mesh resolution (defined in section III.C.1) in order to capture the flow characteristics in more detail.
The distributions of the surface pressure $p$ and the sine of the wall slope angle $\theta$ are plotted in Fig. 23. It can be found that substantial thrust from the inviscid force is attained by the optimum geometries which maintain higher surface pressure levels by moderate expansion over a larger extent, as compared to the baseline geometries in both fuel-on and fuel-off cases. The distributions of the shear stress $\tau$ and the cosine of $\theta$ are plotted in Fig. 24, which shows greater shear stress hence viscous drag experienced by the optimum geometries as a compensation of milder expansion.

It can be noted that the optimum contour in the fuel-on case is featured by maximum $\sin \theta$ and minimum $\cos \theta$ near the shock impingement position (seen in Fig. 22), where both the surface...
pressure and shear stress increase locally. This geometry with a distinct initial arc (seen in Fig. 13) has been produced as a result of the optimization, achieving the greatest possible thrust by maximizing the benefit from the local pressure rise and minimizing the viscous penalty from the shear stress in Eq. (2). The optimum contour in the fuel-off case, by contrast, is characterized by relatively flat surface without a discernible initial arc, which may well be attributed to both downstream occurrence of shock impingement due to shallow shock angles and rather minor influence of the shock impingement on the surface pressure.

C. Performance

1. Total Axial Force

The total axial force achieved by the optimized geometry is plotted for all design conditions in Fig. 25, where the dependency of the force values on computational meshes is also examined for the following resolutions: coarse (35,850 cells), fine (145,095 cells), and superfine (291,015 cells). The mesh density is increased by the factor of two in both circumferential and wall-normal directions for the fine mesh, and by two in the circumferential and four in the wall-normal direction for the superfine mesh, as compared to the coarse mesh, maintaining the minimum cell width normal to the wall surface. Close agreement can be seen between the coarse and superfine meshes, with the maximum deviation found to be within 1%, justifying the choice of the coarse mesh as the nominal resolution in the present optimization study.

The cross-referencing values of the total axial force are plotted in Fig. 26 to investigate the performance of the optimum geometries under off-design conditions. It is noticeable that the geometries optimized for a fuel-on condition are capable of producing comparable forces at the off-design altitudes with fuel-on, and so is the case for the fuel-off optimum geometries, demonstrating the robustness of the optimum designs in cruise conditions at the same state of fuel at different altitudes. This robustness over a range of cruise conditions may well be attributed to the shear layers downstream of the rear base dividing the internal and external flows, which allow the nozzle to perform rather insensitively to the pressure variation associated with the altitude change, as seen in the pressure distributions shown in Fig. 21. The pressure inside the nozzle increases with the ambient pressure (as also seen in the pressure profiles of the nozzle inflow in Fig. 4) as a nature of airbreathing propulsion, in contrast to rocket engines whose performance is highly sensitive to the altitude change due to fixed pressure of nozzle inflow coming out of the combustion chamber.

2. Force Breakdown

The inviscid and viscous contributions of each surface component are plotted in Fig. 27 for the optimum and baseline geometries at 27 km. The total axial force is primarily composed of the inviscid force (thrust) acting on the inner nozzle surface and the viscous force (drag) on the nozzle and external walls, whereas the inviscid force on the rear base has insignificant influence. It is noteworthy that the viscous penalty incurred by extended nozzles, which was discussed in Sec. III.B.2, is well compensated by the appreciable thrust gain, owing to the inviscid nozzle force with the optimum geometries for both fuel-on and fuel-off cases.

3. Conical Nozzles

The sensitivity analysis performed in Sec. III.A.3 has revealed the dominant effects of the nozzle length \( l_n \) and radius \( r_n \) on the total axial force \( F_x \) and its inviscid and viscous components. Following this observation, additional design optimizations have been performed for conical nozzle geometries represented by focusing only on these two parameters, namely \( l_n \) and \( r_n \), which are employed as the sole design variables with the same variable ranges as those used in the preceding study for contoured nozzles. The optimal conical nozzle geometries achieved after 30 generations are plotted in Fig. 28 in comparison with the optimum contoured nozzles for the fuel-on and fuel-off cases at 27 km. The breakdown of the axial force components is compared between the optimized contoured and conical nozzle geometries in Fig. 29. Somewhat less thrust is generated by the inviscid nozzle force with the optimum conical nozzle in the presence of fuel, as compared to that produced by the optimum contoured nozzle, but the loss is compensated to some extent by the appreciable thrust gain from the inviscid force with the optimum contoured nozzle.
extent by the reduction in the viscous drag due to a shorter nozzle. The force components are found to be very similar between the optimized contoured and conical nozzles in the absence of fuel, as a matter of course due to the high resemblance between the two geometries.

Presented in Fig. 30 are the contours of the total axial force with respect to the nozzle length and radius based on all the true CFD evaluations performed in the course of the optimizations, in comparison with the standard 15-deg conical nozzle, which is widely used due to its favorable balance between weight, length, and performance as well as the ease of manufacture [22]. They exhibit similar trends to those obtained for contoured nozzles (Fig. 15) except that the peak (optimum solution) of the conical nozzle has a somewhat smaller nozzle length $l_n$ in the fuel-on case, compared to the optimum contoured nozzle, as also seen in Fig. 28. The divergent half-angle of the optimum geometry is 16 deg in the presence of fuel, whereas it is 10.5 deg in the fuel-off case. The distributions of the inviscid and viscous forces are plotted in Figs. 31 and 32 for the fuel-on and fuel-off cases, respectively. Similar characteristics to those observed in Figs. 17 and 18 are found in the contour patterns, reassuring the rather minor influence of the inner nozzle curvature. It is also noteworthy that the patterns for the viscous force are remarkably similar in Figs. 17b and 31b as well as Figs. 18b and 32b, underpinning the additive nature of the viscous force with respect to the nozzle length $l_n$ and radius $r_n$, as found in the sensitivity analysis conducted in Sec. III.C.

4. Effects of Nozzle Length

The peaks of the force contours plotted in Fig. 30 have indicated that a particular nozzle length can yield the greatest total axial force for a given radius in both fuel-on and fuel-off cases. Parametric studies have been performed by varying the nozzle length while fixing the radius at the optimum value (close to the upper bound of the
variable range, i.e., \( r_n = 0.138 \) m in order to scrutinize the effects of the length on the axial force components. Fig. 33a) indicates that inviscid force (nozzle pressure force plus a minor contribution from the base) increases monotonically to an asymptotic value with the increase of the nozzle length, whereas the viscous drag (skin friction on the nozzle surface and exterior) shows a nearly linear rise in proportion to the length in the fuel-on case at 27 km. The optimum nozzle length is determined by the balance of these counteracting effects so that the sum of their gradients becomes zero at the optimum value. Similar trends are observed in the inviscid force for the fuel-off case plotted in Fig. 33b, except that the inviscid nozzle thrust increases more steadily than the fuel-on case with the increase of the nozzle length, which consequently leads to a larger optimum nozzle length in the fuel-off case than in the fuel-on case (as seen in Fig. 13) as well as the sensitivity of the fuel-off optimum geometry compared to the fuel-on optimum (as also discussed in Sec. III.A.4). Another notable difference is that the inviscid force saturates and levels off once the length exceeds a certain value \( l_n \geq 0.8 \) m, in agreement with the trend observed in Fig. 32a. The viscous drag increases in a linear manner, similarly to the fuel-on case. This tendency observed in the viscous drag is in accordance with the longitudinal (vertical) contours seen in Figs. 31b and 32b.

5. Effects of Nozzle Radius

The influence of the nozzle radius on the thrust is investigated in comparison with analytical estimation. Figure 34 shows the nozzle thrust \( T_n \) due to the force acting on the inner nozzle surface with respect to the nozzle radius \( r_n \), normalized by the combustor radius \( r_c \) for all solutions evaluated in the course of optimization runs along with the ideal curves for the fuel-on and fuel-off conditions at 27 km. The ideal nozzle thrust has been obtained by applying the momentum balance theory to the stream thrust [21] of the nozzle inflow and outflow in conjunction with the area-Mach number relations, making the assumptions that a) the nozzle flow expands isentropically in the chemically frozen state, and b) the exit flow is uniform and parallel without non-axial momentum components. The plots include the values of the optimized conical nozzles for both conditions along with the optimum value of the contoured nozzle in case the nozzle is allowed to expand to a larger radius with the constraint for the minimum base height reduced to 5 mm instead of the original value of 40 mm for the fuel-on case (denoted by a triangle). In both cases, the optimum values can be found to lie closely below the analytical curve prescribed by Eq. (6), which suggests that the benefit in thrust gain diminishes as the nozzle exit radius increases (and hence the nozzle is extended). The difference in thrust between the theory and CFD can be attributed to physical aspects that are not taken into account in theory, i.e., a) entropy increase due to shock waves, skin friction, and wall heat transfer, b) chemical non-equilibrium effects, and c) non-uniformity and non-axial components of the exit gas.
Fig. 30  Total axial force contour plots with respect to nozzle length and radius for conical nozzles (27 km).

Fig. 31  Inviscid and viscous axial force contour plots with respect to nozzle length and radius for conical nozzles (27 km, fuel-on).

Fig. 32  Inviscid and viscous axial force contour plots with respect to nozzle length and radius for conical nozzles (27 km, fuel-on).

Fig. 33  Axial forces for nozzle length variations of optimum conical nozzles (27 km).
An axisymmetric scramjet nozzle configuration including a rear base and external surface has been designed in a population-based optimization approach for cruise conditions at Mach 8 at two altitudes, 27 and 32 km. The geometries have been optimized for maximum total axial force in the presence and absence of fuel by applying evolutionary algorithms assisted by surrogate prediction coupled with computational fluid dynamics. The results have been probed by using various methods including variance-based sensitivity analysis, contour plots, and analytical prediction in order to gain insight into the behavior of the scramjet nozzle flowfields in various aspects, such as the effects of the design parameters, the characteristics of the optimum geometries, and the influence on performance. The insight gained into the flow behavior and performance of axisymmetric scramjet nozzle as a result of the present study is summarized below.

The sensitivity analysis has revealed that the nozzle thrust is determined primarily by the nozzle length and exit radius with minor interactive contributions of the other parameters to the inviscid force. It has been found that the optimum nozzle in the fuel-on case is characterized by bell-shaped contours to make the most of the shock wave entering from the combustion chamber, whereas the fuel-off optimum design features somewhat longer nozzles with rather flat surface, similar to conical nozzles. The optimum nozzle length is determined in relation to the radius to strike the balance between the pressure force and viscous drag, the former of which increases asymptotically while the latter increases linearly with the length. The maximum possible thrust increases asymptotically with the nozzle radius.

Optimum conical nozzles have been found to produce comparable thrust levels, 2 and 0.2% less than the optimum contoured nozzles in the fuel-on and -off conditions, respectively. Cross referencing of the optimum nozzle design in different conditions has shown that the geometries optimized for one altitude can exert equivalent performance at the other altitude at the same Mach number, demonstrating the robustness of the optimum nozzle design under off-design conditions. It is attributed to similar shock structures present in the flowfields at both altitudes due to the adjustment of the internal nozzle pressure in accordance with the ambient pressure that occurs as a nature of airbreathing propulsion, in contrast to rocket nozzles whose inflow pressure is fixed. This feature offers a considerable advantage to scramjet nozzles over rockets, making the performance less susceptible to the altitude change in cruise at a constant Mach number.

### IV. Conclusion

An axisymmetric scramjet nozzle configuration including a rear base and external surface has been designed in a population-based optimization approach for cruise conditions at Mach 8 at two altitudes, 27 and 32 km. The geometries have been optimized for maximum total axial force in the presence and absence of fuel by applying evolutionary algorithms assisted by surrogate prediction coupled with computational fluid dynamics. The results have been probed by using various methods including variance-based sensitivity analysis, contour plots, and analytical prediction in order to gain insight into the behavior of the scramjet nozzle flowfields in various aspects, such as the effects of the design parameters, the characteristics of the optimum geometries, and the influence on performance. The insight gained into the flow behavior and performance of axisymmetric scramjet nozzle as a result of the present study is summarized below.

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