Effects of Leading-Edge Truncation and Stunting on Drag and Efficiency of Busemann Intakes for Axisymmetric Scramjet Engines

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Abstract

Designing high-performance air intakes is of crucial importance for the successful operation of hypersonic scramjet propulsion. This paper investigates the performance of axisymmetric intakes obtained by applying two shortening methods, namely, leading-edge truncation and stunting (longitudinal contraction), to the full Busemann intake in inviscid and viscous flowfields. The primary aim is to identify the key design factors and underlying flow physics in order to achieve the optimum performance with minimum weight by striking the balance between viscous and shock losses. The effects of intake shortening on performance are similar for the two methods for moderate length reduction (25%), conducing to considerable reduction in intake weight. Even reduced to half length, truncated intakes can produce reasonable compression (60% of the full Busemann intake) with the original level of total pressure recovery maintained in viscous flowfields. Stunted intakes, on the other hand, can enhance compression with considerable total pressure penalty, eventually leading to intake unstart at a certain point due to the emergence of Mach reflection, which makes this method potentially useful in situations where locally high pressure and temperature are desired near the centreline at the cost of total pressure recovery.

Key words: Busemann Intake, Hypersonic Airbreathing Engines, Scramjet Performance, High-Speed Aerodynamics, Parametric Studies

1. Introduction

Hypersonic airbreathing propulsion offers great potential for reliable, reusable and economical systems for access-to-space and high-speed atmospheric cruise for both civilian and strategic purposes. Scramjet (supersonic combustion ramjet) propulsion\(^{(1)}\), in particular, is a promising technology that can enable efficient and flexible transport systems, having marked significant milestones in the last decade: the world’s first in-flight demonstration of supersonic combustion via scramjet technology conducted by The University of Queensland (UQ) in the HyShot II Program in July 2002\(^{(2)}\), the fastest flight record via airbreathing engines attained by NASA’s X-43A scramjet-powered vehicle in the Hyper-X program at Mach 9.6 in November 2004\(^{(3)}\), and the longest scramjet burn duration of over 200 seconds achieved in a recent flight of the Boeing X-51A WaveRider in May 2010\(^{(4)}\). An axisymmetric configuration is currently being explored in the SCRAMSPACE project\(^{(5)}\) led by UQ, due to the advantages that the simple geometric configuration can bring about in numerous aspects in-
including aerodynamic and combustion efficiency, aerothermal and structural management as well as manufacture, incorporated with innovative concepts including inlet fuel injection and radical-farming shock-induced combustion\(^{(6,7)}\).

Fig. 1 Full Busemann intake for \(M_{\infty} = 8\) and inviscid flowfields from Taylor-Maccoll equation (top) and computational fluid dynamics (bottom)

Scramjet operation typically depends on flow processing consisting of three stages: airflow capture and compression through the intake, fuel injection and mixing with air in the combustor, and expansion of the reacted gas in the nozzle to produce thrust. The air intake plays a crucial role in this sequential process, being responsible for the compression of inflow to high pressure and temperature required for combustion, while maintaining high total pressure. The full Busemann intake\(^{(8)}\) can achieve high compression in a contracting flow passage in the inviscid flowfield, attaining a total pressure recovery of 97\% with a minimum loss mainly through isentropic compression followed by a single conical shock wave as the only source of an entropy rise\(^{(9)}\), as seen in Fig. 1. However, as any other intake that seeks to minimise leading-edge shock losses, it inherently features an extremely long geometry hence heavy structure and high viscous losses, which subsequently reduce the total pressure recovery to 43\% for the full Busemann intake. An examination of the flowfield shows that the leading-edge surface of the full intake contributes little to compression. These findings encourage the present study involving two representative methods for shortening the full Busemann intake with primary intention to find an optimum intake length that can minimize the sum of boundary-layer and shock losses, striking the balance while maintaining the performance.

This paper presents the results obtained from parametric studies performed numerically for leading-edge truncation of the Busemann intake as well as an alternative shortening method, namely, stunting (contraction in the axial direction). The performance is assessed with respect to various criteria required for scramjet intake design aiming at high compression and total pressure recovery with minimum structural weight. The computational results are analysed in order to extract physical insight and key design factors by scrutinising the inviscid and viscous flowfields for representative cases and comparing with analytical correlations derived from the conservation of mass flow and momentum.

2. Nomenclature

\[
\begin{array}{ll}
\theta_{le} & : \text{leading-edge angle [°]} \\
\Delta L & : \text{length reduction [m]} \\
L_{\text{full}} & : \text{length of full Busemann intake [m]} \\
A & : \text{cross sectional area [m}^2]\text{]} \\
A_{\text{wet}} & : \text{wetted area [m}^2]\text{]} \\
V & : \text{internal volume [m}^3]\text{]} \\
\alpha & : \text{contraction ratio (≡ } A_1/A_2) \\
\eta & : \text{total pressure recovery (≡ } p_{02}/p_{01}) \\
\pi & : \text{pressure ratio (≡ } p_2/p_1) \\
\tau & : \text{temperature ratio (≡ } T_2/T_1) \\
\theta & : \text{property at intake entrance} \\
\theta_2 & : \text{property at intake exit} \\
\theta_\infty & : \text{property of freestream} \\
x & : \text{streamwise coordinate [m]} \\
r & : \text{radial coordinate [m]} \\
p & : \text{static pressure [Pa]} \\
p_0 & : \text{total pressure [Pa]} \\
\rho & : \text{density [kg/m}^3]\text{]} \\
T & : \text{static temperature [K]} \\
T_0 & : \text{total temperature [K]} \\
R & : \text{specific gas constant [J/kg·K]} \\
\gamma & : \text{specific heat ratio} \\
M & : \text{Mach number} \\
U & : \text{velocity [m/s]} \\
C_d & : \text{drag coefficient} \\
D & : \text{intake drag [N]} \\
F & : \text{stream thrust [N]} \\
\alpha & : \text{property at intake entrance} \\
\theta_2 & : \text{property at intake exit} \\
\theta_\infty & : \text{property of freestream}
\end{array}
\]
3. Approaches

3.1. Flow conditions and configurations

The present study focuses on the internal flowfield in the axisymmetric scramjet intake. The captured airflow is a uniform freestream at Mach 8 with a static pressure and temperature of 1197 Pa and 226.5 K, respectively, assuming scramjet operation at an altitude of 30 km. The Reynolds number based on the nominal inlet exit radius of 0.1 m is $1.79 \times 10^6$. The contour for the full Busemann intake is obtained analytically from the Taylor-Maccoll equation\(^{(10)}\). The intake-shortening factor indicates the degree of shortening, defined as $\Delta L/L_{\text{full}}$, that is, the length reduction normalised by the full Busemann length. The contraction ratio decreases steadily from 11.2 with truncation, whereas it remains constant with stunting. The truncated intake geometry is sized for the constant entrance radius of 0.335 m to ensure constant mass flow capture for fair comparison (this leads to the difference in the length of the intakes shortened by the two methods for the same shortening factor). Figure 2 displays the full Busemann geometry with intakes shortened to two representative lengths by the two methods.

The stream-thrust averaging technique\(^{(11)}\) is employed to convert two-dimensional data into scalar values for the evaluation of flow properties such as the Mach number ($M$), pressure ($p$, $p_0$), temperature ($T$), so as to allow quantitative assessment (the flow properties that appear in the discussion and plots in the Results §4 and Appendices §6 are thus stream-thrust quantities unless specified). This paper presents the full results obtained from inviscid and viscous parametric studies for academic interest, where the Busemann intakes are shortened by means of truncation and stunting as long as computations are permissible, while excessive shortening (typically $\Delta L/L_{\text{full}} \geq 0.5$) is of little use due to inadequate compression in practice.

3.2. Computational fluid dynamics

The inlet flowfields are computed by utilising a commercial high-fidelity code CFD++\(^{(12)}\), which solves the Navier-Stokes equations implicitly with second order spatial accuracy. An advanced wall-function technique is used for near-wall treatment and turbulence is modelled by the two-equation SST $k-\omega$ RANS model\(^{(13)}\). The airflow is treated as calorically perfect gas and the inlet surface is assumed to be adiabatic. The outflow is taken to be fully supersonic due to small divergence downstream of the inlet exit. A commercial grid generator Pointwise\(^{(14)}\) is used to generate two-dimensional structured meshes composed of 55,000 cells (276 nodes along the wall and 201 nodes in the wall-normal direction) with the minimum cell thickness on the wall of $10^{-5}$ m in the case of viscous computations (Fig. 3), based on a mesh sensitivity study performed for a previous research dealing with a similar class of intakes\(^{(15)}\).

![Fig. 2 Geometries of full Busemann, truncated and stunted intakes (not to scale)](image)

![Fig. 3 Computational meshes for full Busemann intake (inviscid / viscous flowfield)](image)
4. Results

4.1. Inviscid flowfields

Steady inviscid flowfields have been computed in parametric studies where the shortening factor is varied from 0 (full Busemann intake) to 0.99 in leading-edge truncation and to 0.43 in stunting with an increment of 0.01 (stunted intakes unstart when \( \Delta L/L_{\text{full}} \) is greater than 0.43). The variations of the intake performance in terms of the drag, compression, and total pressure recovery are plotted for both shortening methods in Fig. 4, along with those of the geometric characteristics such as the contraction ratio and leading-edge angle.

Figure 5 displays the effects of shortening on the (stream-thrust averaged) Mach number and temperature at the intake exit as well as the correlations between the pressure ratio and total pressure recovery and those between the exit temperature and drag coefficient, in comparison with theoretical curves derived from the conservation of mass and momentum for adiabatic flow as outlined in the Appendices §6. The flowfields in the truncated and stunted intakes are visualised in Fig. 6 with respect to the Mach number distributions. Figure 7 displays the variations of the temperature profiles at the exit in the course of shortening via stunting.

4.1.1. Drag and compression

The drag coefficient \( C_d \equiv D/(\rho_0 L^2 A_1/2) \) is compared for the two shortening methods in Figure 4(a) compares, where the intake drag \( D \) originates solely from the surface pressure in inviscid flowfields. Truncation results in a drag variation that remains fairly constant within moderate degree of shortening \( (\Delta L/L_{\text{full}} \leq 0.5) \) and decays rapidly with further shortening, whereas stunted intakes are characterised by constant drag increase. These tendencies can be attributed to augmented flow compression in stunted intakes due to increased concave curvature, which is affected less by leading-edge truncation, as seen in the flowfields visualised in Fig. 6. The pressure ratio \( p_2/p_1 \) varies in a very similar manner to the drag coefficient \( C_d \) in the case of stunted intakes, while that of truncated intakes is not characterised by the same tendency but appears to be linked closely with the contraction ratio \( A_1/A_2 \) (Fig. 4(b)). Theoretical reasoning for these trends is given below.

4.1.2. Total pressure recovery

The variations of the total pressure recovery \( p_{02}/p_{01} \) are plotted in Fig. 4(c), where the dependency of the total pressure recovery on the leading-edge angle \( \theta_L \) is apparent for truncated intakes, whereas \( p_{02}/p_{01} \) decreases fairly linearly in stunting. Equation (3) derived in §6.1 indicates that the ratio of the total pressure recovery \( \eta \) \((\equiv p_{02}/p_{01})\) to the pressure ratio \( \pi \) \((\equiv p_2/p_1)\) is a function represented solely by the ratio of the contraction ratio \( \alpha \) \((\equiv A_1/A_2)\) to the pressure ratio \( \pi \). This may well explain the tendency of the pressure ratio \( \pi \) in association with the contraction ratio \( \alpha \) observed above for truncated intakes, by contrast with the stunted intakes, for which the contraction ratio \( \alpha \) is constant, reducing the pressure ratio \( \pi \) to a function only of the total pressure recovery \( \eta \) according to Eq. (3). Figure 5(b) shows the correlations of the pressure ratio \( \pi \) and total pressure recovery \( \eta \) calculated from CFD results, in comparison with \( \eta \) from analytical prescription with Eq. (3) for given values of \( \pi \) evaluated in the results of CFD.

4.1.3. Exit temperature

The variations of the stream-thrust averaged temperature \( T_2 \) evaluated at the intake exit are plotted in Fig. 5(a), along with those of the stream-thrust averaged Mach number \( M_2 \) at the exit as a result of shortening by both methods, where the temperature \( T_2 \) variations are represented by similar curves to those seen for the pressure ratio \( p_{2}/p_{1} \) (Fig. 4(b)). Figure 5(c) displays the evident correlations between the exit temperature \( T_2 \) and drag coefficient \( C_d \) obtained from both CFD and theory (Eq. (9) derived from the momentum balance in §6.2), which has also been observed in a preceding intake optimisation study\(^{(15)}\). The stunted intakes are characterised by abrupt emergence of Mach reflection when the length reduction exceeds a certain value \( (\Delta L/L_{\text{full}} \geq 0.33) \), as confirmed by the presence of a triple point followed by a near-axis subsonic region in Fig. 6(b) and the bulge in the exit temperature profiles plotted in Fig. 7. It is noteworthy that smooth variations are maintained across \( \Delta L/L_{\text{full}} = 0.33 \) in the exit temperature (Fig. 5(b)) as well as in all other stream-thrust averaged properties despite this drastic change in the flow structure.
Fig. 4 Variations of the drag coefficient and static / total pressures in intake shortening (inviscid computations)
Fig. 5 Variations and correlations of the drag coefficient and static / total pressure with the exit Mach number and temperature in intake shortening (inviscid computations)
Fig. 6 Comparison of Mach number distributions between truncated and stunted intakes (inviscid computations)

(a) $\Delta L/L_{full} = 0.2$

(b) $\Delta L/L_{full} = 0.4$

Fig. 7 Variations of exit temperature profiles for stunted intakes (inviscid computations)
4.2. Viscous flowfields

Steady flowfields including viscous effects have been obtained from CFD by varying the intake shortening factor from 0 to 0.99 in truncation and to 0.52 in stunting (which has resulted in unstart when $\Delta L/L_{full} > 0.52$ in viscous simulations). The variations of the intake performance including the breakdown of drag components are plotted for both shortening methods in Fig. 8, along with the geometric characteristics. Figure 9 displays the variations of the viscous drag in relation to the wetted area, and the correlations between the pressure ratio and total pressure recovery as well as the exit Mach number and temperature. Intake flowfields are visualised in terms of the Mach number distributions in Fig. 10 for representative cases and the variations of the exit temperature profiles are displayed in Fig. 11 for stunted intakes.

4.2.1. Drag and wetted area

The variations of the overall drag and its inviscid and viscous components are plotted in Fig. 8 (a). It is observed that both the overall drag and its inviscid component from both shortening methods commonly follow moderately decreasing trends up to $\Delta L/L_{full} \leq 0.2$ with a relatively mild degree of shortening (Figs. 10 (a) and (b)), unlike the inviscid case where the intake drag increases monotonically for stunted intakes (Fig. 4 (a)). This suggests that the presence of the boundary layer may provide some buffering effects for the stunted intakes against increased convex curvature. Beyond this length the stunted intake undergoes rapid rise as a result of curved shock waves, eventually leading to the occurrence of Mach reflection, as indicated by the presence of a Mach disk at the centreline in Fig. 10 (c). The viscous drag decays steadily at similar rates in both methods along with the wetted area plotted in Fig. 9 (a). The wetted area can be regarded as a measure of weight reduction in the assumption that the intake surface is made of constant thickness material. It can be seen that the wetted area decreases at similar pitches with both methods of shortening, indicating similar degree of weight reduction achieved by both methods.

4.2.2. Compression and total pressure recovery

The variations of the pressure ratio are compared in Fig. 8 (b) along with the contraction ratio. Nearly identical variations are seen for shortening up to about $\Delta L/L_{full} = 0.2$, which distinctly differs from the inviscid case, where the two curves for the pressure ratio depart immediately with shortening (Fig. 4 (b)). The pressure ratio increases rather rapidly beyond $\Delta L/L_{full} = 0.2$ in the stunting case due to the formation of stronger shock waves as a consequence of coalesced Mach waves that are compressed on the highly concave intake surface, as seen in Fig 10 (c). The total pressure recovery variations plotted in Fig. 8 (c) indicate that intake stunting yields total pressure recovery slightly higher than truncation for $\Delta L/L_{full} \leq 0.2$, but decays quickly with further shortening. The truncated intakes, on the other hand, are characterised by a rather mild variation of the total pressure over a large extent up to $\Delta L/L_{full} \leq 0.5$ (further truncation beyond this point is impractical due to inadequate flow compression at any rate). Figure 9 (b) exemplifies that the analytical correlation between the pressure ratio $\pi$ and total pressure recovery $\eta$ (Eq. (3) in §6.1) holds in the presence of viscous effects as well.

4.2.3. Exit temperature

The profiles of the exit temperature $T_2$ are plotted in Fig. 11 for stunted intakes. The presence of the thermal boundary layers can be seen in the near-wall region at the ceiling, and the protuberance observed near the centreline with intense stunting (large $\Delta L/L_{full}$ values) can be ascribed to Mach reflection, which gives rise to highly non-uniform flow at the exit. Figure 9 (c) compares the variations of the drag coefficient $C_d$ with respect to the exit temperature $T_2$ calculated by CFD in the course of shortening, along with the theoretical variations prescribed by Eq. (9) derived in §6.2. It is notable that the analytical correlation precisely holds between $C_d$ and $T_2$ in viscous flowfields with adiabatic intake surface, while heat transfer across the cold isothermal surface of 300 K would account for drag increase of an order of 15% for the same exit temperature, according to the results from a preceding study (15) conducted for Buseman-type intakes with similar geometric features and operating conditions. The parameters based on stream-thrust quantities for stunted intakes exhibit rather smooth variations in Figs. 8 and 9 in spite of the abrupt change of the flow structure associated with the emergence of Mach reflection, similarly to the inviscid case.
Fig. 8 Variations of the drag coefficient and static / total pressures in intake shortening (viscous computations)
Fig. 9 Variations and correlations of the drag coefficient and static / total pressure with the exit Mach number and temperature in intake shortening (viscous computations)
Fig. 10 Comparison of Mach number distributions between truncated and stunted intakes (viscous computations unless specified)

Fig. 11 Variations of exit temperature profiles for stunted intakes (viscous computations)
5. Conclusions

Numerical research has been conducted for Busemann intakes shortened by two methods, namely, leading-edge truncation and stunting, aiming at the application to an axisymmetric scramjet engine operating at Mach 8. Parametric studies with various intake lengths have been performed by means of computational fluid dynamics for truncated and stunted intakes in inviscid and viscous flow, in order to investigate the key factors and flow physics for Busemann-based intake design that strikes the optimum balance of shock and viscous losses while incurring minimum structural weight. The performance of shortened intakes has been assessed with respect to various criteria including the drag, pressure ratio, total pressure recovery, wetted area, and exit temperature.

In inviscid flowfields, truncation has resulted in a relatively constant level of intake drag within a practical degree of shortening up to about 50%. The total pressure recovery undergoes moderate reduction of approximately 25% at half length, in close coordination with the leading-edge angle as well as in concordance with an analytical correlation with the pressure and contraction ratios, both of which vary rather gradually. On the other hand stunted intakes feature fast enhancement of compression at the expense of greater drag and total pressure losses due to shock formation, which leads to intake unstart when reduced by more than 43%. The emergence of Mach reflection which triggers unstart, however, gives rise to locally high temperature in the vicinity of the centreline owing to the presence of a Mach disk without radically affecting other global flow quantities and performance parameters, rendering this shortening method potentially suitable for applications that require high pressure and temperature particularly near the centreline.

Viscous simulations have revealed considerable influence of the boundary layers on the intake performance and flowfields. The total pressure recovery, which is reduced to less than half of the inviscid value due to viscous losses for the full Busemann geometry, remains little affected by shortening with truncation up to half length, in accordance with the observation made in Reference (16), while it experiences slight gain with stunting up to 20% reduction in length but deteriorates rather quickly until intake is unstarted with a length reduction of 52% in the viscous case. The skin friction drag decreases constantly as the intake is shortened with both methods, and so does the inviscid drag component, but stunting leads to rapid increase in the inviscid and overall drag as well as compression once the shortening factor exceeds 20% due to the formation of strong shock waves as a result of intense compression in a rapidly contracting flow passage. The exit temperature varies smoothly in correlation with the overall drag as per theoretical prescription, even in the presence of Mach reflection which causes substantial non-uniformity in the exit flow profile.

In summary leading-edge truncation will serve as a useful technique to reduce the intake length and hence its structural weight (according to the wetted area) down to half of the full Busemann geometry without incurring substantial penalty in the intake performance in terms of drag, compression and total pressure recovery. Stunting, on the other hand, will be as useful as truncation within length reduction of 20%, beyond which the incident occurrence of Mach reflection with this method may offer advantages in limited circumstances where a localised high temperature core is desired to be present near the centreline in the design requirement.

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6. Appendices

6.1. Correlation between total pressure recovery and pressure ratio

By coupling the conservation of mass \((p_1 U_1 A_1 = p_2 U_2 A_2)\) and the equation of state \((p = \rho RT)\) as well as the definitions of the Mach number and speed of sound, we have:

\[
\frac{T_2}{T_1} = \left(\frac{p_2 M_2 A_2}{p_1 M_1 A_1}\right)^2 \tag{1}
\]

For adiabatic flow that is assumed in the present study, the total energy hence total temperature is constant throughout the inlet compression, thus:

\[
\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^\pi \tag{2}
\]

By equating Eqs. (1) and (2) as well as \(T \equiv T_0/(1 + \frac{\gamma - 1}{2} M^2)\), the following relationship can be derived between the total pressure recovery, pressure ratio, and contraction ratio:

\[
\eta \equiv \frac{1}{2} \left\{ \frac{1}{2 \beta_1} + \sqrt{\left(\frac{1}{2 \beta_1}\right)^2 + \frac{\gamma - 1}{2 \beta_1} \left(\frac{\pi}{\alpha}\right)^2 M_1^2} \right\} \tag{3}
\]

where \(\eta\) is the total pressure recovery \((\equiv p_{02}/p_{01})\), \(\pi\) the pressure ratio \((\equiv p_2/p_1)\), \(\alpha\) the contraction ratio \((\equiv A_1/A_2)\), and \(\beta_1 \equiv 1 + \frac{\gamma - 1}{2} M_1^2\).

6.2. Correlation between intake drag coefficient and exit temperature

By using the notations introduced above and the temperature ratio \(\tau \equiv T_2/T_1\), Eq. (1) can be rewritten as:

\[
\tau = \left(\frac{\pi M_2}{\alpha M_1}\right)^2 \tag{4}
\]

For adiabatic flow the temperature ratio \(\tau\) can also be written as:

\[
\tau = \frac{T_2}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 = \frac{\beta_1}{1 + \frac{\gamma - 1}{2} M_2^2} \tag{5}
\]

Elimination of \(M_2\) from Eqs. (4) and (5) yields:

\[
\frac{\pi}{\alpha} = \sqrt{\frac{\gamma - 1}{2}} M_1 \tau \sqrt{\frac{1}{\beta_1 - \tau}} \tag{6}
\]

The momentum balance in the control volume allows the intake drag \(D\) to be expressed as the difference in the stream thrust \(F \equiv (p + \rho U^2/2)A\) between the entrance and exit:

\[
D = F_1 - F_2 = (p_1 A_1 + p_1 U_1^2 A_1) - (p_2 A_2 + p_2 U_2^2 A_2) \tag{7}
\]

The drag coefficient \(C_d\) is given by non-dimensionalising the drag \(D\) with \(p_1 U_1^2 A_1/2\):

\[
C_d = \frac{2}{\gamma M_1^2} \left\{ (1 + \gamma M_1^2) - (1 + \gamma M_2^2) \frac{\pi}{\alpha} \right\} \tag{8}
\]

Substituting \(M_2\) and \(\pi/\alpha\) from Eqs. (5) and (6), respectively, into the above equation, the drag coefficient \(C_d\) can be expressed by the following formula comprising the temperature ratio \(\tau\) and other parameters that are constant for fixed air capture and calorically perfect gas:

\[
C_d = \frac{2}{\gamma M_1^2} \left\{ (1 + \gamma M_1^2) - \left( \tau \sqrt{\frac{\gamma - 1}{2(\beta_1 - \tau)}} + \frac{2(\beta_1 - \tau)}{\gamma - 1} \right) M_1 \right\} \tag{9}
\]
References


